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Impact of Power Control on Capacity of TDM-scheduled Wireless Mesh Networks *

Parth H. Pathak, Rudra Dutta
Department of Computer Science, North Carolina State University, USA. Email: phpathak@ncsu.edu, dutta@csc.ncsu.edu

***Abstract*—In this paper, we consider power control as network layer problem in wireless mesh networks. The network connectivity between nodes is determined by their communication range which in turn can be controlled by adjusting the transmit power level. It is generally acknowledged that reducing transmit power levels of nodes to the minimum required to retain connectivity always increases network capacity. In this work, we show that though this is true for CSMA/CA based medium access, increasing power level of nodes can be beneficial in many cases when links are TDM-scheduled. Based on analysis and simulations, it is observed that increasing power levels of nodes (and decreasing number of hops in routing paths) results in increase of throughput in many representative traffic patterns and topologies. We characterize achievable spatial reuse and capacity with respect to power control in different topologies and traffic patterns. With increasing number of MAC protocols adopting TDMA approach, results presented here can be crucial in understanding how capacity is affected with varying levels of network connectivity.**

I. Introduction

Wireless Mesh Networks (WMNs) have gained increasing popularity as an alternative for low cost and high speed wireless access networks. The backhaul tier of WMN experiences relatively stable and heavy traffic load due to multi-hop relaying of data between mesh routers. In the current deployments of WMN, it has been observed that 802.11-based CSMA/CA MAC does not scale well with increasing traffic demand due to packet collisions and its conservative nature of collision avoidance. This has lead to research and development of various TDMA based medium access protocols where collisions are avoided by scheduling interfering links in different time slots. Few of the examples of such protocols include IEEE standards like 802.11s [1], 802.16 [2] and MAC protocols which are built on top of existing 802.11 hardware/software [3], [4]. It has been also demonstrated [5], [3] that time divided link scheduling outperforms CSMA/CA in terms of achievable throughput in WMNs. One of the outstanding challenges in implementation of TDMA based MAC has been time synchronization among the nodes. To deal with this issue, researchers have designed coarse-grained TDMA scheduler where each time slot is significantly larger than the time required for the transmission of a packet. In such case, when a particular link is scheduled for transmission in a slot, communicating pair of nodes exchange many packets during that time. The advantages of such a scheme are its feasibility of implementation [6] and analytical tractability [5] [7]. The problem of link scheduling (also referred as spatial reuse TDMA) with time divided multiplexing is to schedule given transmission demand of links in minimum number of slots (maximize throughput). Based on the interference relationship between links, only a subset of links can be scheduled in every slot.

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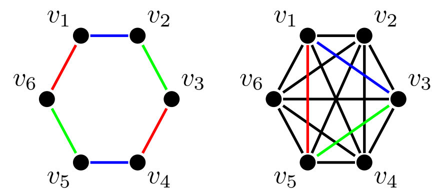

Fig. 1: (a) Compow graph (b) DirectTrans graph

When CSMA-CA based MAC is employed, increasing transmission power level of nodes results into more interference and collisions, which in turn reduces the throughput capacity. This is generally acknowledged due to extensive literature on power control [8] and topology control [9], which shows that increasing transmission power of nodes results into larger collision domains and due to random access nature of CSMA, overall spatial reuse reduces significantly when the transmissions are not separated in frequency or time domain. We show that in TDM-scheduled WMNs, throughput behavior depends on *range-hop* tradeoff of various power control strategies. The trade-off is best characterized by two contradicting power control strategies (not specific to TDMA) proposed by *short-range-multi-hop* Compow [10] and *long-range-single-hop* DirectTrans [11]. In Compow, all nodes use a uniform constant power level which is minimum required to maintain network connectivity. Compow achieves better concurrency in link scheduling but requires more relaying at nodes with longer routing paths. In sharp contrast to this, in DirectTrans, nodes willing to transmit increase their power level until receiver can be reached in single hop. DirectTrans benefits from fewer transmissions and no relaying but suffers with lower spatial reuse due to long high interference links.

Fig. 1 shows an example which demonstrates the trade-off. Assume that there are three flows $v_1 \rightarrow v_3, v_3 \rightarrow v_5, v_5 \rightarrow v_1$, each transferring one unit of traffic. Also, assume that shortest path routing is used and $2-$hop interference model determines sets of mutually interfering links. In Compow topology (left), all the six links have to be scheduled once to satisfy traffic demands of three flows. Links of the same color represent a set of links which have to be scheduled once and do not mutually interfere. Routing path for every flow in this case contains two hops. When TDM scheduler is used to schedule the links, three slots are required to complete the transmission demands since two links can be scheduled in each of the three slot (e.g. v_1v_2 and v_4v_5). On the other hand, in DirectTrans topology (right), all nodes are directly in one hop reach of all other nodes (clique). In this case, only one transmission is required for every flow but each of the transmission interferes with all other transmissions (e.g. v_1v_3). The spatial reuse in this case drops to one, also requiring three slots for satisfying traffic demands. This way, even though Compow and DirectTrans are two extremes of uniform power control, their performance can become comparable in TDM-scheduled WMNs.

In this paper, we study the shorter-hops-longer-paths and longer-hops-shorter-paths tradeoff in terms of attainable ca-

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pacity with different traffic patterns and topologies. It is shown that complex interplay of spatial reuse and total number of required transmissions determine the actual throughput at various levels of connectivity between nodes. Surprisingly, in many representative traffic patterns and network topologies, increase of power levels results in increased throughput, which is in sharp contrast to the CSMA/CA case.

Some of the important results of the paper are as follows:

1) In random topologies under uniform node-to-node traffic, minimally connected Compow and fully connected DirectTrans achieve almost equal throughput capacity. (Sec. III-A1)
2) When network topology is even slightly clustered, increasing power levels of nodes results into better throughput (DirectTrans outperforms Compow). We show that *transitivity* of the Compow graph of any point pattern can be used to estimate the behavior of throughput capacity with increase of power levels. (Sec. III-A2)
3) In the highly realistic and practically important node-to-gateway traffic pattern, we show that increasing power level of the nodes always results into better throughput. We prove that WMNs achieve per node throughput of $O(1/\delta n)$, where δ is a factor dependent on hop-radius of the network, which in turn is dependent on power level of nodes (Sec. III-B).

These results provide first insights about behavior of capacity with power control in different topologies and traffic patterns when links are TDM-scheduled.

II. Network Model and Methodology

We model the network graph using unit disk graph $G_U = (V, E, \lambda)$ where V is the set of nodes and for any two nodes u and v, there exists an edge $uv \in E$ if their Euclidean distance $d_{uv} \leq \lambda$. Compow range (d_{min}) is defined as minimum value of λ such that G_U is connected. Similarly, DirectTrans range (d_{max}) is minimum value of λ such that G_U is *fully* connected (clique). We refer to the Compow graph of V as $G_C = (V, E_C, d_{min})$ and DirectTrans graph as $G_D = (V, E_D, d_{max})$.

As we stated, our principal problem is to investigate the impact of changing power levels of nodes on throughput capacity in different topologies and traffic patterns. We approach the problem by using uniform power control method of increasing power levels and communication range. In uniform power control, all nodes increase their power levels step-by-step by a common factor such that at any step, they all operate at the same power level. We use path loss model of signal propagation. If transmitted signal power is P_t and distance between the transmitter and the receiver is d then received signal power (P_r) attenuates as $P_r \propto P_t(d^{-\alpha})$, where α is the path loss exponent which depends on environment ($2 \leq \alpha \leq 5$). Let β be the receiver sensitivity threshold such that signal is properly decoded at the receiver if $P_r \geq \beta$. The communication range of a node is the distance at which $P_r = \beta$ in absence of any other interference. Now, power level of nodes can be presented in terms of their communication range. As an example, in G_C all nodes are operating at power level $P(d_{min})$ which is necessary and sufficient to achieve communication range of d_{min} at all nodes.

During the uniform increase of power levels, every node increases its communication range by a factor of f from the Compow range. This is achieved by tuning its power level to $P(f \cdot d_{min})$. This way, increase of power levels are normalized to the Compow range (d_{min}), not to the Compow power level ($P(d_{min})$). We refer to f as the *growth factor* of connectivity. In all cases, two widely used traffic patterns namely, *uniform node-to-node* (Sec. III-A) and *uniform node-to-gateway* (Sec. III-B) are studied. In uniform node-to-node traffic pattern, every pair of source and destination communicate with amount of traffic which is uniform across all such pairs. In uniform node-to-gateway traffic, all nodes send uniform amount of traffic to the gateway only. Also, since the study pertains to wireless mesh networks (not energy-constrained), nodes do not have any restrictions on available power resources. Without loss of generality, we assume that all nodes operate on the same channel. Problems like channel assignment, physical layer rate adaptation etc. are orthogonal to the work presented here.

III. Throughput Capacity

TDMA-based link scheduling problem as described above is known to be NP-hard for protocol interference model [12], $K-$hop interference model (where $K \geq 2$) [13] and SINR-based physical interference model [7]. The focus of this work is not to device an efficient scheduling algorithm but to understand the effects of power control on capacity when the links are TDM-scheduled. Hence, we use greedy link scheduler for generating time slotted, conflict-free and feasible link transmission schedule and compare its performance with bounds derived on optimal schedule. Such greedy scheduling algorithm was proposed in [5] and [14] for physical and protocol interference model respectively. As we will see, the performance of the greedy scheduler is within a constant factor from the optimal schedule.

First, we briefly describe the TDMA greedy link scheduler [14] and then proceed with the discussion of throughput capacity in various cases. The end-to-end traffic demand between nodes is represented using a traffic demand matrix (T_R). Once the shortest path routing is performed, T_R yields per-link transmission matrix (T_X). We assume that there is a central controller entity which performs link scheduling. In the operation of greedy scheduler, first all links of T_X are sorted based on their interference score. Interference score of a link is the number of other links with whom the given link interferes and hence can not be scheduled simultaneously. Then scheduler chooses the first link in order to be scheduled in the current slot and tries to add more and more non-interfering links greedily until no more links can be added to the slot. The procedure repeats until all transmission requests of T_X are satisfied. [5] showed that such a scheduler has the time complexity of $O(m \cdot n \cdot T)$, where $T = \Sigma_{i=0}^{n} \Sigma_{j=0}^{n} T_{X_{ij}}$. If the total offered load $G = \Sigma_{i=0}^{n} \Sigma_{j=0}^{n} T_{R_{ij}}$ and greedy scheduler requires S slots to schedule all the links, the network throughput is G/S traffic units per unit time.

We assume that simultaneous transmissions on two links uv and xy results into collision-free data reception at the receivers *iff* $d_{ux}, d_{uy}, d_{vx}, d_{vy} > (\Delta \cdot d_{uv})$ and $d_{ux}, d_{uy}, d_{vx}, d_{vy} > (\Delta \cdot d_{xy})$, where d_{xy} is the distance between nodes x and y and interference ratio $\Delta = 2$. Such a consideration of interference in greedy scheduler allows us to focus on network connectivity without delving into physical layer details such as physical layer data rate, signal quality etc. Using this knowledge, now we move on to the capacity analysis and results for different traffic patterns and topologies. First, we consider a more general case of uniform node-to-node traffic and then look at WMN specific uniform node-to-gateway traffic pattern.

A. UNIFORM NODE-TO-NODE (N2N) TRAFFIC

In all cases, growth factor f is used to uniformly increase the connectivity such that resulting network graph transforms from a minimally connected Compow graph (G_C) to completely connected DirectTrans graph (G_D). It is worth acknowledging that the power level required to achieve DirectTrans range at each node might be very high and practically impossible. We still include this extreme case in study because step-by-step transformation of network graph from Compow to DirectTrans facilitates various intermediate levels of connectivity which are practically feasible and provide better understanding of general behavior of throughput capacity.

1) Uniform Topology: When the nodes are distributed with uniform density in the plane, transmission *footprint* based analysis can be used to calculate the average spatial reuse. Though this is a simplification of actual disk packing based analysis, we will see that the proposed method yields close approximation of theoretical capacity.

Let l denote the length of the side in a square network area and let G be the total offered load (N2N), that is $G = \Sigma_{i=0}^{n} \Sigma_{j=0}^{n} T_{R_{ij}}$. Let $\lambda = d_{min}$ denote the Compow range for the given node placement. At every state of uniform power increase, communication range ($f\lambda$) and interference range ($\Delta f \lambda$) of nodes also increase. Based on the interference model, every link transmission occupies a wireless footprint of an area worth an overlapping double disk. Such a double disk is created by having one disk of radius $\Delta f \lambda$ at each end points of the link. Now, the best case of spatial reuse can be considered as tightly packed double disks of links in the network area, each with radius of $f\lambda$ only. This is when end points of two active links have overlapping interference ranges but their communication ranges do not overlap. With the radius of $f\lambda$, expected value of area of such overlapping double disk of every link is approximately $5(f\lambda)^2$. No other transmission can be concurrently scheduled in this area without causing interference to ongoing transmission. Thus, average spatial reuse in any slot can be approximated by (network area / expected value of area of double disk) = $\frac{(l+2f\lambda)^2}{5(f\lambda)^2}$. Considering $(l + 2f\lambda)$ for the calculation of network area allows better consideration of link transmissions over the boundary of the network at least when $l \gg f\lambda$.

Let L denote the average distance between any source-destination pair in the network. With the employment of shortest path routing and assumption of uniform node density, average path length in terms of hops can be approximated as $\frac{L}{f\lambda}$. Total number of link transmissions required to satisfy offered load G is then $T = \frac{GL}{f\lambda}$. Combining the results of average spatial reuse and expected value of total number of link transmissions T, expected number of slots S required to satisfy G is $\frac{5GLf\lambda}{(l+2f\lambda)^2}$.

Note that for any particular offered load, changing the connectivity among the nodes changes the number of slots required to schedule the transmissions. As shown above, when f increases, total number of required transmissions (T) to satisfy G decreases. On the other hand, spatial reuse also decreases with the increase in value of f. This way, T and the value of spatial reuse determines the required number of slots S. In the above mentioned equation of S, the values of L, l and λ are dependent on node distribution and network area, which in turn shows that required number of slots has a clear dependence on value of the growth factor f.

Fig. 2a, Fig. 2b and Fig. 2c show analytical (proven above) and simulated values of throughput capacity, spatial reuse and T respectively. The noticeable difference between the two curves in Fig. 2a can be attributed to following two reasons. First, the analytical values derived above attempt to approximate the optimal performance while the simulation results are obtained using greedy scheduler. Second, the above mentioned analysis requires the network to be highly dense for the correct approximation of average path length using $\frac{L}{f\lambda}$. Here, nodes are placed as a 20 $\times$ 20 square grid with offered load being uniform node-to-node traffic. For increasing value of growth factor, we apply equal amount of traffic G to the resultant graph. This can be interpreted as transition from Compow graph to DirectTrans graph and capacity at each stage of growth. The data is routed on a shortest path which is closest to the straight line connecting the source and the destination (e.g. straight line routing [15]).

As can be observed, initial increase in power level of nodes decrements the throughput (number of required slots increase) followed by an increment. This can be characterized as follows - increase in power levels increments the link lengths which in turn decreases spatial reuse T. Decrement in the spatial reuse (Fig. 2b) is much more faster than that in T (Fig. 2c). So, in this initial region, lower spatial reuse is achieved even when very high number of transmissions are required, increasing the total number of required slots to schedule them. When T also further decreases, this again results into increase of the throughput. Note that even a minor difference between theoretical and simulated values of spatial reuse (or T) can cause significant difference between theoretical and simulated values of S as observed in Fig. 2a. In the later part (for $13 \leq f \leq 19$ in Fig. 2a), the throughput capacity in simulation remains unchanged. This is due to the fact that majority of the links at these states of uniform power increase are long, high interference links (loner links [14]) which make the spatial reuse to drop to one in almost all slots. Area based formulation fails the capture this behavior which results into monotonous decrease of S in analytical curve during this later region.

We now compare the simulated throughput of greedy scheduler with upper bound of optimal throughput (lower bound on optimal schedule length) which is derived using conflict graph. A conflict graph is created by placing a vertex for every link of communication graph and an edge between two vertices if corresponding links in communication graph interfere with each other. We use traffic demand of every link in communication graph as the weight of the corresponding

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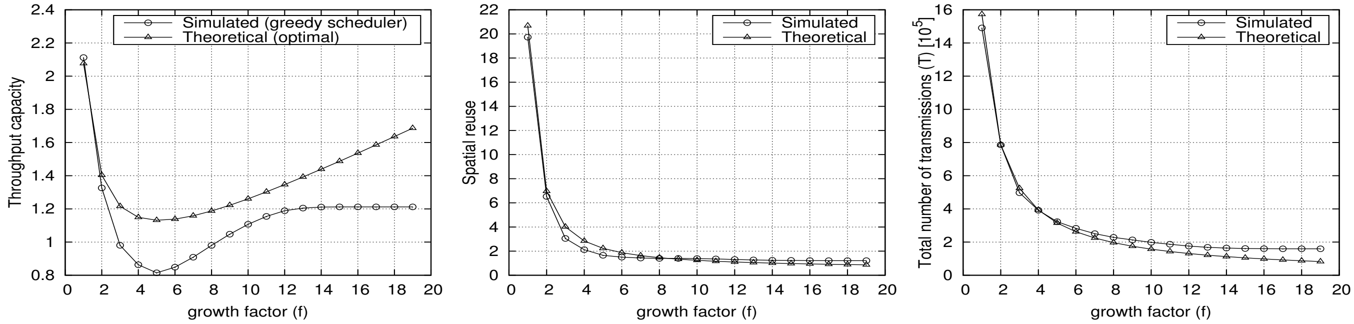
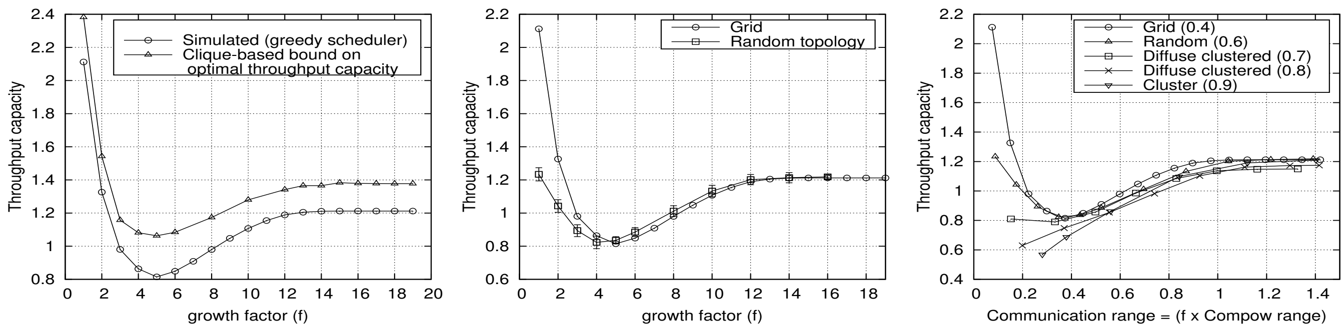


(a) Performance of links scheduling in grid (n=400) (b) Achieved spatial reuse while scheduling T_X (c) Total number of transmissions required to satisfy G



(d) Upper bound on throughput capacity (e) Greedy scheduling in random topologies (f) Throughput and topology transitivity (n=400)

Fig. 2: Effects of power control on link scheduling with node-to-node uniform traffic pattern

vertex in the conflict graph. Now, maximum weight *clique* in the conflict graph is a set of links in communication graph which has maximum total traffic but only one of which can be scheduled in any slot. In most cases, links of nodes near the center (relay hot-spot) form such higher traffic cliques. Since no two links in such a set of mutually interfering links can be scheduled in the same slot, even optimal scheduler requires number of slots which is at least the size of the clique (in terms of traffic). Numerical results of this upper bound on throughput capacity are presented in Fig. 2d. An interesting observation here is that in all cases, greedy scheduler achieves the throughput which is no worse than half the optimal throughput.

Fig. 2e shows the throughput results of greedy scheduler for random topologies. Homogeneous Poisson point processes are used to generate random topologies with complete spatial randomness. The Compow range of random node placement is typically higher than that of the grid placement for same number of nodes due to spatial non-uniformity. The simulations are repeated for 30 random topologies and corresponding confidence intervals are presented. *Here, it is interesting to note that in fact Compow and DirectTrans (two extremes of power control) achieve almost equal capacity under uniform N2N load in random topologies.* This shows that multi-hop routing on almost minimally connected sparse graph of Compow and direct transmissions in fully-connected DirectTrans topology yield equivalent capacity when links are TDM-scheduled. This is attributed to the interplay of attainable spatial reuse and total number of required transmissions.

2) Clustered Topology: Most of the real-world topologies actually display some degree of *clusteredness*. They are either highly clustered or fall between random topologies and clustered topologies. We refer to such topologies as diffuse clusters. Since the Compow range is the minimum common range such that the nodes are connected, it increases as the topology becomes more and more clustered. This results into higher intra-cluster connectivity but inter-cluster connectivity still remains low. In such case, links providing connectivity between the clusters become traffic bottlenecks under N2N traffic. These links require large number of transmissions and consume large number of time slots when link scheduling is performed. This increases the overall schedule length and reduces the attainable capacity. When power is increased uniformly, new inter-cluster links are added which share the traffic burden and reduces the traffic bottlenecks in the network. This results into higher spatial reuse among the links and overall throughput capacity increases. Increasing power levels of nodes in such case actually has positive effect on throughput. This is surprisingly different from uniform topologies where increasing power levels in N2N traffic had mix effects on throughput.

For study of connectivity, clustering coefficient based on spatial pattern of node distribution is not enough. Instead, we use graph clustering coefficient proposed in [16] which is also known as *transitivity* of a graph. Let $\Delta(v)$ denote number of triangles which have v as one of its three vertices and $\nu(v)$ denote number of triples of vertex v. A *triple* is a connected subgraph which has three vertices but only two edges. A triple belongs to a vertex v if v is the vertex which has both edges incident to it. Now, number of triangles in graph G, $\Delta(G) = 1/3\Sigma_{v\in V}\Delta(v)$ while number of triples in graph G, $\nu(G) = \Sigma_{v\in V}\nu(v)$. Then transitivity of the graph G is defined as:

$$\tau(G) = \frac{3\Delta(G)}{\nu(G)} \quad (1)$$

Thus, transitivity $(0 \leq \tau(G) \leq 1)$ can be interpreted as likelihood that any two neighbors of a node are connected. As an example, for a given set of points, a spanning tree has transitivity of 0 while complete graph has transitivity of 1. It can be seen that if the Compow range of set of points is relatively high, the Compow graph of points is expected to have higher transitivity. Note that Compow graph is a unique graph for any particular set of points. Compow graphs of several point patterns have following τ values (approximately): perfect square grid - 0, perturbed square grid - 0.2, square grid with diagonal - 0.44, Poisson random - 0.6, diffuse clusters - 0.7-0.8 and clustered - 0.9.

Transitivity of any Compow graph shows an interesting relation with how throughput changes when power is increased uniformly. This is shown in Fig. 2f. We simulate clusters of different intensities using Mateŕn cluster process [17]. Results are presented using greedy scheduler with 400 nodes deployed on a unit square. Throughput changes its behavior differently as transitivity increases from a perfect grid to clustered topologies. In topologies which have transitivity similar to diffuse cluster or more, in fact increasing power levels always results into better throughput. Clique based upper bound of optimal scheduler also follows the same behavior, results of which we do not show for brevity. This shows that for the same traffic pattern (node-to-node uniform), uniform random topology and clustered topology results into different throughput behavior with change in power levels. *This is a practically useful result since it shows that increasing power levels of nodes results into better throughput capacity when the nodes are clustered. Also, for a given set of nodes, transitivity of the Compow graph can be calculated and then it is possible to predict the behavior of throughput with increase of power levels.*

B. UNIFORM NODE-TO-GATEWAY (N2G) TRAFFIC

Now, we analyze a more commonly used and practically important traffic pattern which is N2G traffic. The most representative use of such traffic pattern is observed in Wireless Mesh Networks (WMNs) in which all nodes send traffic to one or more gateway nodes which forwards it further to the Internet. In one of the most useful results about capacity of WMNs, [18] proved that per node throughput in WMNs can not be more than $O(1/n)$ which is significantly worse than classical result of $O(1/\sqrt{n})$ presented in [19]. This is because traffic of every node, no matter how many hops away it is from the gateway, has to ultimately traverse through the bottleneck links connecting to the gateway. Per node throughput of $O(1/n)$ is also achievable in WLANs, but it has been empirically observed that WMNs often achieve even worse throughput than WLANs (often by a factor 2 or 3).

We show that WMNs in fact achieve per node throughput of $O(1/\delta n)$, where $\delta \geq 1$ is a factor dependent on the hop-radius of the network graph. Here, *hop-radius* is defined as length of the longest path from a node to the gateway assuming that the gateway has the least eccentricity among all nodes. The result pertains to networks where interference model is based on the distance. Below we present the analysis for achievable per node throughput in WMNs with bi-directional protocol interference model (as described in Sec. III). We then go on and show how this throughput capacity is dependent on the power assignment.

1) Analysis: As before, let λ denote the Compow range, and power levels of all nodes are incremented using the growth factor f resulting into communication range of $f\lambda$ at every node. Let R denote the hop-radius of the network graph. Let tier t be a set of nodes that are t hops away from the gateway and hence R is also the number of tiers in the network. With assumptions of uniform and dense distribution of nodes, number of tiers in the network $R \approx \frac{R_N}{f\lambda}$ where R_N is the Euclidean radius of circular network area into consideration. Now, number of nodes in any tier i can be given as :

$$n_i = \frac{(2i-1)n}{R^2} \tag{2}$$

Every node in the network sends p packets to the gateway during the time frame under consideration. For simplicity of analysis, a link can be associated with the tier of its source node. Links of any tier i forward traffic for all nodes at tier $i+1...R$ as well as traffic of nodes at its own tier. It can be assumed that shortest paths to the gateway always contain links which forwards traffic between the tiers. With the assumption of uniform density of nodes, load on all links in tier i is:

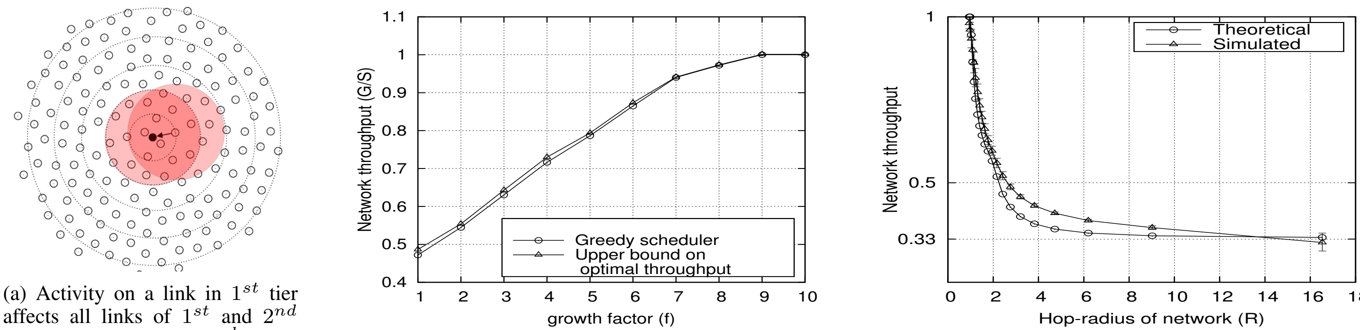
$$L_i = p \cdot (n - \sum_{j=1}^{i-1} n_j) = p \cdot \frac{n(R^2-(i-1)^2)}{R^2} \tag{3}$$

Bottleneck Collision Domain (BCD) is defined as set of mutually interfering links in which no more than one link can be scheduled in same slot and cumulatively they have to transmit maximum total traffic. It is easy to see that in WMNs, such a BCD is often created near the gateway node. As shown in Fig. 3a, when a link in first tier is scheduled, almost all links in second and third tiers suffer from interference and can not be scheduled concurrently. For the worst case performance analysis, we consider that all links of first three tiers are part of single BCD. This can be a really pessimistic consideration of interference since it might be possible to schedule two links concurrently in third tier which are on opposite sides each other (Fig. 3a). We will discuss later how considering the links of only first two tiers in BCD can be too optimistic. The size of BCD ($\mid BCD \mid$) is defined as total amount of traffic its links have to transmit which can be calculated as:

$$\begin{aligned} \mid BCD \mid &= L_1 + L_2 + L_3 \\ &= p \cdot n + p \cdot n \left(\frac{R^2-1}{R^2}\right) + p \cdot n \left(\frac{R^2-4}{R^2}\right) \\ &= p \cdot n \left(\frac{3R^2-5}{R^2}\right) \end{aligned} \tag{4}$$

By definition of BCD, total network traffic np requires at least $\mid BCD \mid$ number of slots. Hence, network throughput can not be more than $\frac{np}{|BCD|}$ and with assumption of absolute fairness, per node throughput can not be more than $\frac{p}{|BCD|}$. Combining this with (4), per node throughput can not be more than $1/\delta n$ where $\delta = \frac{3R^2-5}{R^2}$. Hence, it is shown that per node throughput in WMNs is $O(1/\delta n)$ where δ is a factor which depends on number of tiers in the network.

When $R = 2$, every node can achieve throughput of $\frac{p}{|BCD|}$ where $\mid BCD \mid = L_1 + L_2 = p \cdot n \left(\frac{2R^2-1}{R^2}\right)$. Since $R = 2$, $\delta = \left(\frac{2R^2-1}{R^2}\right) = 1.75$. Similarly, it can be shown that when $R = 1$, $\mid BCD \mid = L_1 = p \cdot n$ and fraction $\delta = 1$. This results into



(a) Activity on a link in 1^{st} tier affects all links of 1^{st} and 2^{nd} tier and many links of 3^{rd} tier

(b) Greedy scheduler performs near optimally in WMNs

(c) Relation of Throughput and hop-radius in WMNs

Fig. 3: Effect of power control on link scheduling in wireless mesh networks

$O(1/n)$ where every node can directly reach gateway similar to the WLAN case.

Surprisingly, factor δ converges to 3 as $R \rightarrow \infty$ in (4). *This suggests that no matter how large the hop-radius of the network becomes, WMNs can still achieve one third of the capacity of WLANs even in the worst case.* This is due to the fact that at least one link in first three tiers can always be scheduled in almost all slots. It is important to note that the value to which the δ converges also depends on how BCD is calculated, which in turn is dependent on the interference model under consideration.

2) Simulation Results: In node-to-gateway uniform traffic pattern, greedy scheduler performs near-optimal because almost all links away from the gateway (beyond first three tiers) can be almost always be scheduled in parallel with links of first three tiers (BCD). Hence, number of slot required $S \approx | BCD |$. This is verified by comparing the performance of greedy scheduler and upper bound of optimal throughput (Fig. 3b). Similarly, Fig. 3c verifies simulated throughput with analytical result of $O(1/\delta n)$. For simulations, 1000 nodes are randomly placed on a unit-disk with the gateway placed at the origin. For achieving continuous values of throughput points, we assume hop-radius $R = \frac{R_N}{f\lambda}$ to be continuous also. As the hop-radius of the network increases network throughput drops from 1 and converges to 0.33 for large values of radius. It is noticeable that even when radius increases from 1 to 2, network throughput drops significantly (by a factor $\delta = 1.75$).

The minor difference between the analytical and simulated curve can be attributed to the fact that average spatial reuse in first three tiers is often slightly larger than 1. This is in line with our rather pessimistic analysis which assumed that only one link can be scheduled in first three tiers. If it would have been assumed that only one link can be scheduled in first *two* tiers, analytical value of throughput would have converged to 0.5 (as shown in Fig. 3c) which would have been clearly too optimistic when compared to actual simulated results.

The analysis shows clear dependence of δ on R which in turn dependent on Euclidean network radius R_N, Compow range λ and growth factor f. Since R_N and λ are dependent on network deployment area and node distribution, growth factor f is the only control parameter of interest. Dependence on f shows that increase of f results in smaller network hop-radius R which reduces factor δ, resulting into actual increase of throughput. *This suggests that it is always better to increase power level of nodes which decreases the network hop-radius and increases the throughput. This is a useful capacity result especially in case of WMNs since it proves that reducing worst case hop distance to gateway always performs better in terms of throughput.* This is also true when multiple gateways [20] are utilized in a WMN.

IV. Conclusions

In this paper, we investigated impact of uniform power control on throughput capacity in different traffic patterns and topologies. We showed that when the links are TDM-scheduled, increasing power levels of nodes results into increased throughput in case of many representative topologies (e.g. clustered) and traffic patterns (e.g. N2G traffic).

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