



## Centrality-based power control for hot-spot mitigation in multi-hop wireless networks <sup>☆</sup>

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### ABSTRACT

When shortest path routing is employed in large scale multi-hop wireless networks such as sensor networks, nodes located near the center of the network have to perform disproportionate amount of relaying for others. Nodes in such traffic hot-spots deplete their batteries faster than others due to their high relay load. These traffic hot-spots also adversely affect the network capacity due to increased congestion in the regions. To solve the problem, various divergent routing schemes are used which route the data on center-avoiding divergent routing paths. Though they achieve better load balancing, overall relaying is increased significantly due to their longer routing paths. In this paper, we propose power control as a way for balancing relay load and mitigating hot-spots in wireless sensor networks. Using a heuristic based on the concept of *centrality*, we show that if we increase the power levels of only the nodes which are expected to relay more packets, significant relay load balancing can be achieved even with shortest path routing. Different from divergent routing schemes, such load balancing strategy is applicable to any arbitrary topology and traffic pattern. With extensive simulations, we show that centrality based power control can drastically increase the network lifetime of sensor networks. We compare its performance with other divergent routing schemes and multiple battery level assignment strategy. Also, it is shown that centrality based power control results into better throughput capacity in many different topologies.

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### 1. Introduction

Many of the routing protocols proposed for multi-hop wireless networks depend on shortest path routing (SPR) due to its characteristics of simplicity, robustness and scalability. It has been observed that when hop count based shortest path routing (SPR) is used for uniform node-to-node communications in a multi-hop network, certain nodes have to perform disproportionate relaying of data for others. Such *hot-spots* are often created near the center in uniform topologies and also at cluster peripheries in clustered topologies. This increased congestion in certain areas has shown to be resulting into reduced network capacity. When the nodes are energy constrained as in sensor networks, nodes performing higher amount of relaying than others deplete their batteries faster, reducing the overall network lifetime. As an example, when traffic flows between random source and destination pairs, network lifetime is often bounded by the lifetime of the nodes near

the center since majority of the end-to-end shortest paths pass through them. On the other hand, when nodes send their data to a central entity (a sink or a gateway), generally one hop neighbors of the sink have to perform the most relaying which results into earlier depletion of their batteries [1], followed by disconnection of the sink from other alive nodes. In most cases, relay load distribution of nodes is significantly unfair where some nodes exhaust their batteries very quickly while others have not consumed even half of their energy resource.

The problem of disproportionate relaying has been addressed mainly by devising routing strategies in which routing paths intentionally try to avoid passing via center. Curved paths in curve-ball routing [2,3], one-turn rectilinear paths in Manhattan routing [4] and edge reflection paths of outer space routing [5] are some examples of such strategies. We refer to such center-avoidance routing strategies generally as *divergent* routing. Any such divergent routing scheme increases relaying load of the nodes near the periphery of the network while taking away some relaying burden from the nodes around the center. This results into better overall load balancing. This advantage comes at a cost of various other sacrifices. Most of the divergent routing schemes depend on geometrical properties of the network (for mapping the network over symmetric space like torus or sphere) which limits their applicability to uniform topologies only. Routing paths in any such scheme

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are also longer (higher stretch factor) when compared to shortest routing paths. Moreover, divergent routing schemes sacrifice the fundamental advantages of SPR such as robustness, scalability and simplicity. In many cases, divergent routing schemes do not eliminate the hot-spots in the network because the relay load of the nodes near the center decreases moderately while the load on nodes around the periphery increases significantly. This results into only a moderate increase of lifetime of network since the relay load is not completely balanced among nodes. This raises an interesting question – is it possible to preserve SPR (and all its advantages) while achieving better relay load balancing?

One of the assumptions in divergent routing schemes is that all nodes operate at Compow [6] power level and routing is performed on the resultant topology. In Compow, all nodes use a uniform power level which is minimum required to maintain network connectivity. Compow achieves better concurrency in link scheduling due to lesser interference but requires more relaying at nodes because of longer routing paths. This motivates the following questions which are central to our work – is disproportional relay load distribution in multi-hop communications also a consequence of Compow power range together with SPR? And can it be dealt with using power control instead of modifying routing?

In this paper, we propose power control as a way to balance relay load in wireless sensor networks. We show that if the communication range of nodes are properly increased by increasing their power levels, better relay load balancing can be achieved even when routing on the shortest paths. In all cases, choosing only a small subset of nodes and increasing their power levels is sufficient to achieve load balancing that is significantly better than divergent routing schemes. The fundamental advantage of such power control based load balancing is that it preserves all the benefits of shortest path routing and still achieves a significant relay load balancing. Because all the characteristics of shortest path routing is retained, such load balancing can be applied to any kind of arbitrary topologies (e.g. clustered) and traffic patterns (e.g. node-to-gateway) where divergent routing can not be applied.

The proposed load balancing scheme does not route the data on divergent routes to avoid passing through the nodes near the center. Instead, the data is routed on the shortest paths only and the nodes which are expected to relay more packets for others are skipped or *jumped over*. In the case of uniform topologies, this results into longer hops being taken near the center which reduces the relay load burden of congested nodes without increasing the relay load of the nodes on the periphery. Power control based load balancing presented here assigns higher power levels to nodes which are expected to relay more packets. This has an underlying requirement of estimating the relay load of nodes in advance so that communication range can be assigned to them accordingly. We calculate *betweenness* centrality of nodes which assigns every node a score based on their expected relay load. The centrality value is then used to assign every node a power level which is proportional to its expected relay load. This increases the connectivity of the nodes who were expected to relay more packets previously. When shortest paths between nodes are found in this new more connected network, they pass over congested nodes, producing better load balancing.

Since the relay load is better balanced among the nodes with actual reduction in overall relaying, the overall network lifetime is improved using centrality based power control. Note that longer hops in routing paths can only be achieved by controlling the communication range of nodes using power control. It may appear at first that since some nodes are performing high power transmissions, the overall energy expenditure might increase with centrality based power control. Surprisingly, the proposed scheme achieves an intelligent balance between the transmission power level assigned to every node and its corresponding relay load. This

also guarantees an improved balance of energy expenditure among the nodes because the power assignment itself is performed using the predicted relay load. We show that improved relay load balancing, reduced overall relaying and evenly distributed energy expenditure increases the network lifetime of sensor networks dramatically. The numerical results show up to 30% increase in network lifetime when compared to divergent routing schemes.

One of the other disadvantages of the divergent routing schemes is that there is a trade-off [7,8] between length of the routing paths (path-stretch) and amount of load balancing achieved. We show that power control based load balancing does not show such a trade-off with path stretch. Instead, it is observed that load balancing and capacity sometimes show a trade-off when power control based load balancing is applied in some of the topologies. We characterize that such a trade-off is limited to the case of uniform node placement and as the node placement becomes more random and clustered, increased capacity can be achieved together with better relay load balancing.

The rest of the paper is organized as follows – we start by explaining network model and related assumptions in Section 2. Section 3 presents centrality based power control strategy and shows how betweenness centrality can be used for power allocation. We present several numerical results on load balancing, network lifetime and throughput capacity in Section 4. We conclude in Section 5 with important remarks about our ongoing work on distributed protocol design which is based on the work presented in this paper.

## 2. Network model and preliminaries

We model the network using a directed graph  $G = (V, E, r)$ , where  $V$  is a set of  $n$  stationary nodes and  $r$  is a set of communication range values assigned to each node in  $V$ . There exists an edge from node  $u$  to node  $v$  if their Euclidean distance  $d_{uv} \leq r_u$ . This allows the modeling of variable power control where each node might have a different communication range. Note that when the value of  $r_v$  is same for all  $v \in V$ , the resultant network graph can be considered as an undirected graph (unit disk graph) since all the edges are bidirectional. In one such case, when there is no explicit power control, all nodes are assumed to be operating at Compow power level. Compow range ( $r_{min}$ ) where  $r_v = r_{min}, \forall v \in V$  is defined as minimum value of common range such that  $G$  is connected. We refer to the Compow graph as  $G_c$ . We have proved that  $r_{min}$  is the value of maximum edge weight in a Euclidean minimum spanning tree of  $V$  (also asserted, but without proof, in [9,10]). The complete proof is presented in Appendix A; this result allows us to efficiently determine  $r_{min}$  for large set of nodes using any known polynomial time algorithm.

Centrality values of all nodes are determined from the Compow graph and are used to assign different power levels to different nodes. Here, increase in power level of a node can actually be interpreted as upper constraint placed on the transmission power level of the node. That is, if a node is assigned a specific power level, this does not mean that it will always transmit at that power level. If a neighbor is reachable at a lower power level, it will utilize that to communicate with it. In this case, each node uses power just sufficient to provide the range to the destination node of each transmission, but cannot exceed the “assigned power level”. Thus some nodes are out of range at any assigned level, but more nodes become available at higher levels. All the schemes presented in the work along with the proposed scheme take advantage of this power control policy. We use path loss model of signal propagation. If transmitted signal power is  $P_t$  and distance between the transmitter and the receiver is  $d$  then received signal power ( $P_r$ ) attenuates as  $P_r \propto P_t(d^{-\alpha})$ , where  $\alpha$  is the path loss exponent which depends on environment ( $2 \leq \alpha \leq 5$ ). Let  $\beta$  be the receiver

sensitivity threshold such that signal is properly decoded at the receiver if  $P_r \geq \beta$ . For a node transmitting at power  $P_t$ , the communication range is the distance at which  $P_r = \beta$  in absence of any other interference. Now, power level of nodes can be presented in terms of their communication range. As an example, in  $G_c$  all nodes are operating at power level  $P(r_{min})$  which is necessary and sufficient to achieve communication range of  $r_{min}$  at all nodes. Now, if a node wants to increase its communication range by a factor of  $f$ , it tunes its power level to  $P(f \cdot r_{min})$ . This way, increase of power levels are normalized to the Compow range  $r_{min}$ , not to Compow power level  $P(r_{min})$  because  $P(f \cdot r_{min}) \neq f \cdot P(r_{min})$  necessarily.

Two widely used traffic patterns namely, uniform node-to-node and uniform node-to-sink are studied in this paper. In uniform node-to-node traffic, every pair of source and destination communicate with amount of traffic which is uniform across all such pairs. Such traffic pattern is commonly used in applications like querying and storing data within the network [11–13], and target tracking [14–18] in military sensor networks. In uniform node-to-sink traffic, all nodes send uniform amount of traffic to the sink only. Habitat monitoring, environment observation and structure health monitoring [19] are some of the applications where such a traffic pattern is commonly utilized. We assume that the nodes are stationary and there is no consideration of mobility in this work. Note that the presented work relates to the throughput capacity of the network, and is not dependent on what application uses that capacity. The results obtained here are most useful for applications in which the traffic being generated by each node is more or less constant, as in many sensor network as well as mesh networks. Without loss of generality, we also assume that all nodes operate on the same channel.

### 3. Centrality-based power control

Conceptual difference between load balancing using divergent routing and power control is depicted in Fig. 1. It can be observed that divergent routing schemes (Fig. 1c and d) transfers much of the relay burden onto the nodes around the periphery. In power control based load balancing (Fig. 1e and f), data is forwarded on shortest path only but nodes in hot-spot regions are skipped or *jumped over*. This results into longer hops taken near the center hot-spot in uniform topologies. To achieve such longer hops, nodes which are expected to relay more packets should be assigned high power levels and larger communication range. If the relay load at every node can be estimated, then a node can be assigned power levels proportional to its estimated load. This increases the connectivity of the nodes which were likely to relay more packets previously. When shortest paths between nodes are found in this new more connected network, they pass over congested nodes, producing better load balancing. The mentioned heuristic has an underlying requirement for accurately estimating the relay probability of nodes for any given topology which we discuss next.

#### 3.1. Modeling relay load

Techniques for modeling and estimating relay load have been limited to uniform topologies with assumptions like continuous density of nodes. One of the first few such approaches was presented in [2] where relay load probability of a node was derived as a function of its (Euclidean) distance from the center of the network, when routing over the shortest paths between the nodes. It is possible to show that nodes at same distance from the center can have different relay load if the topology is even slightly non-uniform. Similarly, [20] presented Voronoi cell based technique in which it was shown that relay load of a node is a function of perimeter of its Voronoi cell, its location in the network and the traffic pattern in consideration. Even with assumptions of uniform node-to-node traffic pattern and node distribution, Voronoi tessellation based model might not always be sufficient because nodes adjacent to each other in network graph might not always have neighboring Voronoi cells. Such techniques do not always accurately estimate the relay load because they rely on geometrical properties of the network and not the underlying network connectivity.

##### 3.1.1. Betweenness centrality

We are interested in devising a relay load estimation technique which relies on properties of network graph. Centrality indices [21] proposed for analysis of large network assign relative importance or status to every node in the network based on certain characteristic of interest. One such centrality named betweenness [22] of a node depends on how many end-to-end shortest paths between other nodes of network actually pass through it. In node-to-node uniform traffic, a node is likely relay more data for others if it falls on relatively more number of shortest paths between other nodes. For a network graph  $G$ , if  $S_{xy}$  is number of shortest paths between vertices  $x$  and  $y$  and  $S_{xy}(v)$  denotes number of shortest paths between  $x$  and  $y$  which pass through vertex  $v$ , then betweenness centrality of  $v$  (denoted by  $b(v)$ ) is

$$b(v) = \sum_{x \neq y \neq v \in V} \frac{S_{xy}(v)}{S_{xy}} \quad (1)$$

The fraction in (1) (also known as *pair-dependency*) can be interpreted as the probability that vertex  $v$  will be relaying data between vertices  $x$  and  $y$ . Betweenness centrality of a node can be regarded as measure of how important a node is in carrying out relaying of data for other nodes.

Betweenness centrality is a structural index of the graph, that is if  $H$  is isomorphic to  $G$  ( $G \simeq H$ ), then betweenness centrality satisfies the condition:  $\forall v \in V : G \simeq H \Rightarrow b_G(v) = b_H(\phi(v))$ . Also, betweenness of nodes change considerably with changes in edge set of network graph which may require frequent recalculations. This is not a concern here since we do not consider any mobility of nodes. As described in [21], straightforward usage of Dijkstra's algorithm for computing betweenness of nodes can have running

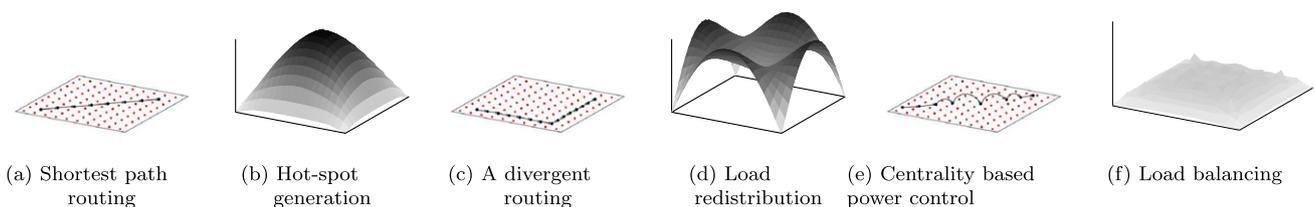


Fig. 1. General nature of path and relay load characteristics.

time of  $O(n^3)$ . Brandes's algorithm [23] can compute betweenness of all nodes in  $O(nm)$  time for unweighted graphs and  $O(nm + n^2 \log n)$  for weighted graphs, where  $m$  is number of edges in the graph. In terms of  $n$ , this does not impose significant computational cost for moderate to large sized networks (400–1000 nodes). While in terms of  $m$ , as it is described below, betweenness centrality is required to be determined in the Compow graph which is a relatively sparser graph ( $m \ll n(n - 1)$ ). When considering clustered topologies, intra-cluster connectivity can be much higher even in COMPOW graphs which can increase make  $m$  large. In such cases, Delaunay triangulation can be used for generation of a sparser graph and calculation of betweenness centrality indices. This can significantly reduce the value of  $m$  and can still maintain a bounded stretch of shortest paths (no more than a stretch of  $\frac{2\pi}{3 \cos(\pi/6)}$  in geometric graphs [24]). Fig. 2 shows betweenness centrality of nodes in Compow graph and Delaunay graph, and compares it with their actual simulated relay load in Compow graph when employing uniform node-to-node traffic pattern and SPR.

### 3.2. Centrality-based power control

Following steps describe how betweenness indices are used to assign power levels to nodes.

1. For a given  $V$ , first Compow range ( $r_{min}$ ) is determined and  $G_C = (V, E_C, r_{min})$  is created.
2. Betweenness centrality of all nodes in  $G_C$  are calculated using Brandes's algorithm and are normalized using  $\max \{b(v) | v \in V\}$ .
3. Every node  $v \in V$  is assigned a power level ( $P_v$ ) as:  $P_v = P(D_{min} + (b(v) \cdot (D_{max} - D_{min})))$  where  $D_{min} \geq r_{min}$  to guarantee connectivity and  $D_{max} \geq D_{min}$ . Even if  $D_{max} = D_{min} = r_{min}$ , resultant graph is at least  $G_C$ .

Any such assignment is uniquely referred as  $\psi(D_{min}, D_{max})$  and resultant more connected graph of betweenness centrality based power assignment is called  $G_B$ . We set  $D_{min} = r_{min}$  and vary  $D_{max} = f \cdot r_{min}$  using a factor  $f \geq 1$ . Here,  $D_{min}$  and  $D_{max}$  are dependent on  $r_{min}$  which is a property of  $V$ , only control parameter is  $f$  which we refer as *growth factor*. For any reasonable value of  $f$ ,  $\psi$  results into increased power levels and communication range of nodes which were expected to relay more packets. Nodes near the center in uniform topologies have higher betweenness and are assigned higher power levels. As shown in Fig. 1f, this allows them to *jump over*

other nearby congested neighbors. If a source and a destination are on opposite side of each other over the periphery, packet from source first starts progressing along shorter hops. As it reaches near the center, long-distance transmissions occur which results into longer hops, followed by fewer shorter hops at the end. This results into subsequent reduction of relay load of nodes near the center without increasing relay burden on nodes on periphery.

Foremost advantage of such load balancing using power assignment is that it can be applied to any kind of arbitrary topology like clustered where divergent routing mechanisms can not be applied. Also, centrality measure of nodes can be calculated for any specific set of shortest paths pertaining to traffic pattern of interest, which makes the mechanism applicable for load balancing in any other traffic patterns (e.g. node-to-sink uniform). The complexity of the scheme is dominated by the complexity of centrality algorithm, and Brandes's algorithm performs well even with moderately large networks as we saw above. The presented scheme is static and centrality calculation is necessary only once. The only necessary requirement of the scheme is that the nodes should be able to vary their power levels as per the centrality based calculations. Note that the proposed power control is centralized and its distributed extensions are discussed in future work.

Fig. 3 shows the relay of nodes when centrality based power assignment is used for load balancing. It shows the impact of growth factor on load balancing in a  $20 \times 20$  grid network. As discussed before,  $D_{min}$  is set to  $r_{min}$  and  $D_{max} = f \cdot r_{min}$ . Initially, when  $f = 1$ ,  $D_{max} = D_{min}$  and resultant topology is a Compow graph of  $V$ . In such case, there is no explicit effect of centrality values because growth factor is set to 1 and relay load distribution displays hot-spots near the center. When growth factor increases, the actual difference between maximum and minimum power level assigned in the network also increases. This results into higher power levels and communication ranges for nodes which have higher betweenness centrality and are expected to relay more packets. As can be observed in Fig. 3, central nodes are now assigned higher power levels which results into better load balancing. The growth factor  $f$  is a tunable parameter here and its value can be decided based on several factor which we discuss later.

### 3.3. Analysis

The load balancing achieved by the centrality based power control depends on the value of the growth factor. Increasing the growth factor also increases the connectivity among the nodes. The relation between the load balancing and connectivity was first established by [7]. To further generalize, let  $H$  be a graph where betweenness centrality of nodes are found, and let  $G$  be the resultant graph once the power assignment  $\psi(D_{min}, D_{max})$  is applied. It is easy to see that edge set  $E_H \subseteq E_G$  and  $H$  is a spanner of  $G$ . Let  $dm_G$  and  $dm_H$  denote the diameter of  $G$  and  $H$  respectively. In fact,  $H$  is referred as  $\sigma$ -spanner of  $G$  where  $\sigma = dm_H/dm_G$  is called the stretch factor of  $H$ .

Now, let us assume that there exist an optimal load balancing routing strategy and  $I^R(u)$  denotes relay load imposed on node  $u$  by the optimal load balancing routing of traffic demand set  $\mathcal{R}$  in a network. Let  $I^R(G) = \max \{I^R(u) | u \in V\}$  and *load balancing ratio* for two different connected graphs  $H$  and  $G$  defined on the same point set  $V$  is  $I^R(H)/I^R(G)$ . [7] proved the following relationship between the stretch factor and the load balancing ratio.

**Lemma 1.** *If  $H$  is  $\sigma$ -spanner of  $G$  then for a given request set  $\mathcal{R}$  and optimal load balancing routing,  $I^R(G)/I^R(H) = O(1/\sigma^2)$ .*

This further formalizes the fact that increasing the value of the growth factor  $f$  decreases the diameter of  $G$  which in turn increases the stretch factor  $\sigma$ . This results into improved load balancing in  $G$

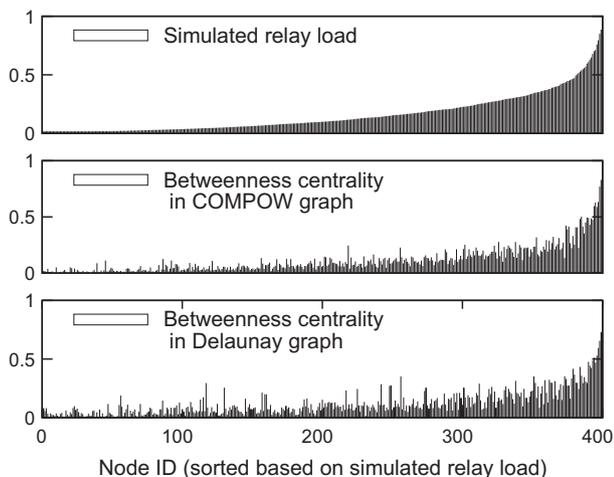


Fig. 2. Comparison of simulated relay load of nodes and their centrality values in Compow graph and Delaunay graph (all normalized with maximum value,  $n = 400$  randomly distributed in unit square).

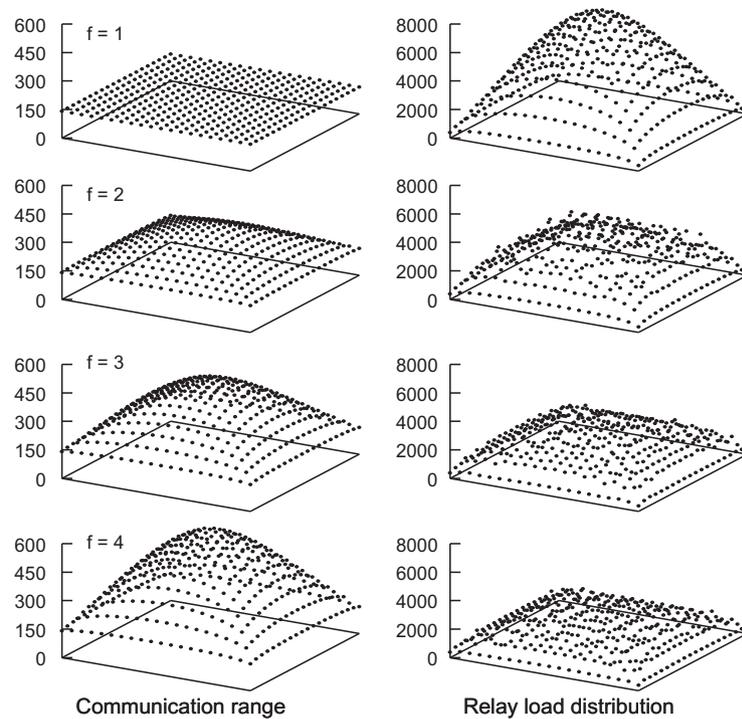


Fig. 3. Relay load of nodes and effect of growth factor on load balancing in a  $20 \times 20$  grid with node-to-node uniform traffic.

as compared to that of its spanner  $H$ . Note that this is proven for an optimal load balancing routing scheme, not the shortest path routing which is central to this work. We demonstrate via simulation results (presented in Section 4) that in the set of representative topologies we study here, it is indeed true that increasing the growth factor  $f$  results into improved load balancing.

### 3.4. Impact on energy efficiency

The centrality based load balancing mechanism increases connectivity between the nodes in the regions which were previously congested. As in Fig. 3 where  $f = 4$ , nodes near the central hot-spot are assigned high communication range which results into reduced overall relaying in the area. It might appear at first that since some nodes in centrality based power control are transmitting at higher power levels and may deplete their batteries faster than other. But in fact number of transmissions required by such nodes are reduced significantly. Centrality based power control tries to achieve a balance between the nodes which have to transmit more number of times but with lower power levels and the nodes having to transmit at higher power levels but lesser number of times. The increase in energy expenditure for any given node is more than offset by the reduction in the transmission load of that node. This way, appropriate balance is achieved between actual amount of relaying and power level of nodes due to proper utilization of their betweenness values.

Apart from better load balancing, centrality based power control reduces the overall energy consumption because of reduction in total number of required transmissions and receptions. Reception is also a significant reason of power consumption in wireless sensor networks. It was shown in [25] that routing on shortest paths with least number of hops is almost always more energy efficient. When all nodes are operating at Compow power level, connectivity among the nodes are lesser than when they operate on centrality based power levels. So, shortest paths in topology resulting from centrality based power control will have lesser number of hops than shortest paths of Compow topology. This way, when

routing uniform node-to-node traffic on shortest paths, centrality based power control requires lesser number of transmissions/receptions than Compow power control mechanism, which in turn results into lower overall power consumption.

Centrality based power control mechanism with shortest paths is also more energy efficient than divergent routing scheme with Compow power levels. Any divergent routing scheme of load balancing results into longer routing paths (more number of hops) when compared to shortest paths. This is typically measured using *average path stretch* of a routing scheme which can be defined as average ratio of hop-length of routing paths in a divergent routing scheme to hop-length of shortest routing paths. The path stretch and load balancing ratio display a trade-off ([7,8]) in any divergent routing scheme which makes better load balancing with lower stretch factor inherently difficult. Hence, load balancing improves with increased path stretch which results into increased path lengths and more number of transmissions and receptions. As an example, it was shown in [5] that outer space routing consumes 1.4 times more energy than shortest path routing. Hence, in general case, divergent routing scheme would end up spending more energy than shortest path routing.

Collectively, centrality based power control mechanism reduces overall energy consumption when compared to divergent routing schemes. Also, the energy expenditure of nodes is also more uniform. This results into drastic overall increase of network lifetime of sensor networks.

## 4. Numerical results

Three sets of simulation results are presented to confirm the claims of load balancing using centrality-based power control. In the first set, load balancing strategy is applied to different kinds of topologies of sensor networks. We also compare its load balancing performance to existing divergent routing schemes. Next, we show how the load balancing affects the attainable capacity in different topologies. At the end, we present energy-efficiency and network lifetime results for sensor networks.

4.1. Load balancing

Fig. 4a shows the relay load of nodes resulted using SPR in  $G_C$  and centrality based more connected graph  $G_B$ . We use  $20 \times 20$  grid and set the growth factor  $f = 6$ . That is, if  $b(v)$  was the betweenness centrality of a node  $v$ , operating at power level  $P(r_{min})$  in  $G_C$ , in  $G_B$ , it is assigned power level  $P_v = P(r_{min} + b(v) \cdot ((6 * r_{min}) - (r_{min})))$ . As expected, relay load distribution in  $G_C$  clearly shows hot-spots near the center. In  $G_B$ , even with shortest path routing, relay load is very well distributed among nodes.

Next, we analyze the effect of growth factor on load balancing achieved by  $\psi(D_{min}, D_{max})$  power control. We consider grid (Fig. 4a), random (Fig. 4b) and clustered topologies (Fig. 4c). Random topologies ( $n \approx 400$ ) are modeled as homogeneous Poisson point processes while clustered topologies ( $n \approx 300$ ) are generated using Matérn cluster process [26]. Shortest path routing is used with node-to-node uniform traffic pattern. In all cases,  $D_{max}$  is varied by the growth factor  $f$  using  $D_{max} = f * r_{min}$  and  $D_{min}$  is set to  $r_{min}$ . Whenever random topologies are considered, results are presented for 30 instances of random topology with 95% confidence interval. Here relay load is defined as number of packets a node has to relay for other nodes. It is observed that even for small growth factor, relay load balancing improves significantly in all topologies. Though initial decrease in maximum, average and standard deviation of relay load is more significant, it consistently decreases with higher values of  $f$ . In clustered topologies, nodes providing inter-cluster connectivity become traffic bottlenecks and resultant relay load distribution can be highly disproportionate.

based load balancing schemes are not applicable in such clustered topologies. As can be observed,  $\psi$  achieves significantly better relay load balancing in clustered topologies even for very small values of  $f$ .

Now, we compare the  $\psi$  with other well-known divergent routing schemes. Load balancing of shortest path routing in centrality based power controlled topology ( $G_B$ ) is compared with the load balancing of shortest path routing in Compow topology ( $G_C$ ), and three well-known divergent routing schemes, namely outer space routing [5], Manhattan routing [4] and curve-ball routing [2]. In outer space routing, packets are first forwarded towards the periphery of the network and is then reflected back from some intermediate node towards the actual destination. In Manhattan one-turn routing, source forwards the packet to an intermediate node which is near the intersection of horizontal/vertical lines passing through the source and destination. In curve-ball routing, network plane is first mapped on a sphere and shortest paths on sphere are then mapped back to the plane. This results into center-avoiding curved routing paths. All divergent routing schemes are also employed in  $G_C$ .

Fig. 5a shows the results of load balancing in a 400 nodes network and uniform node-to-node traffic pattern. We use growth factor  $f = 6$  in  $\psi$  as described in Section 3.2. Centrality based power control mechanism achieves significantly better load balancing compared to SPR in  $G_C$  and divergent routing schemes. Divergent routing schemes increase the average relay load of nodes when compared to SPR but decreases the overall deviation, which shows better load balancing. On the other hand,  $\psi$  decreases the standard

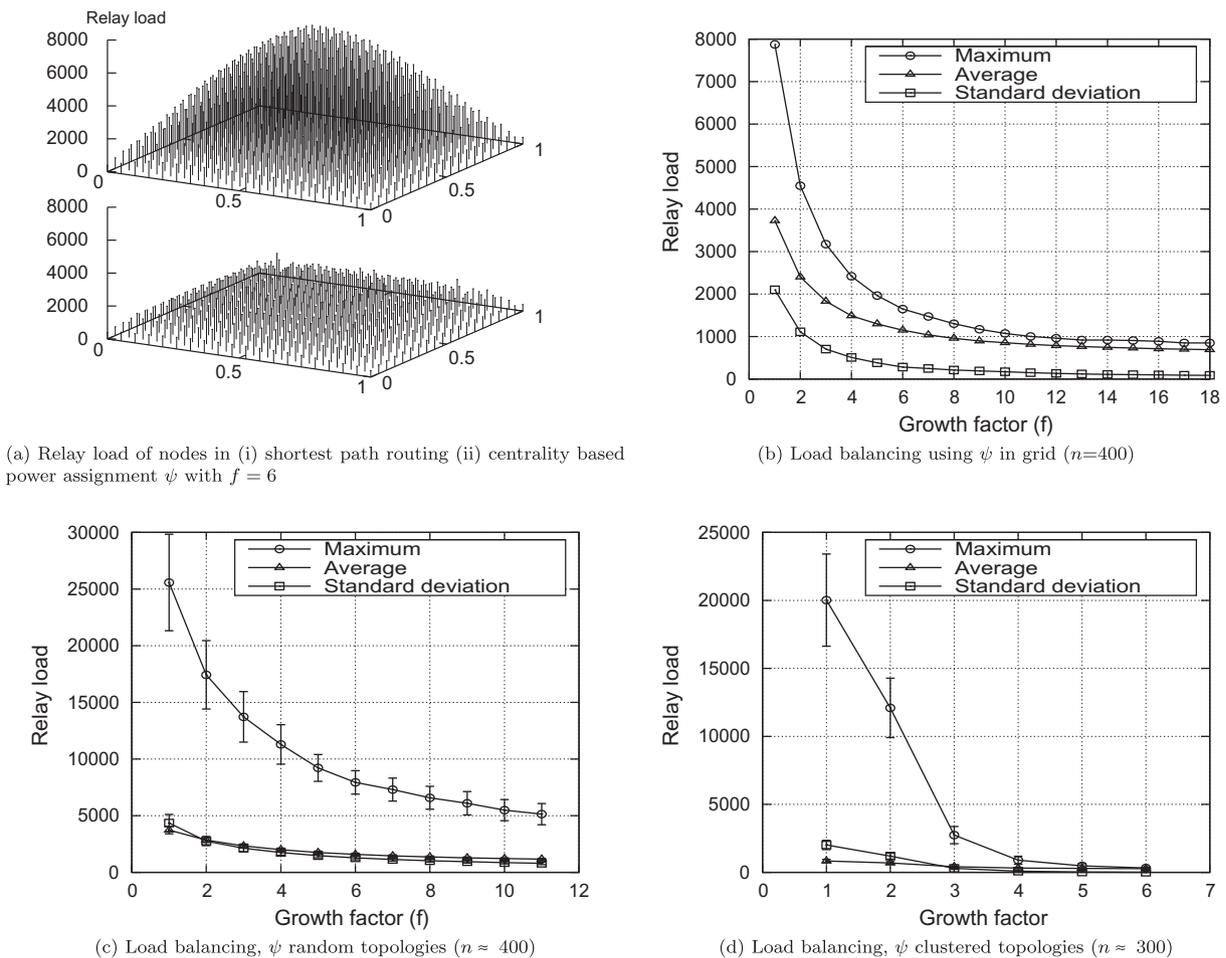


Fig. 4. Load balancing effects of centrality based power assignment  $\psi$ .

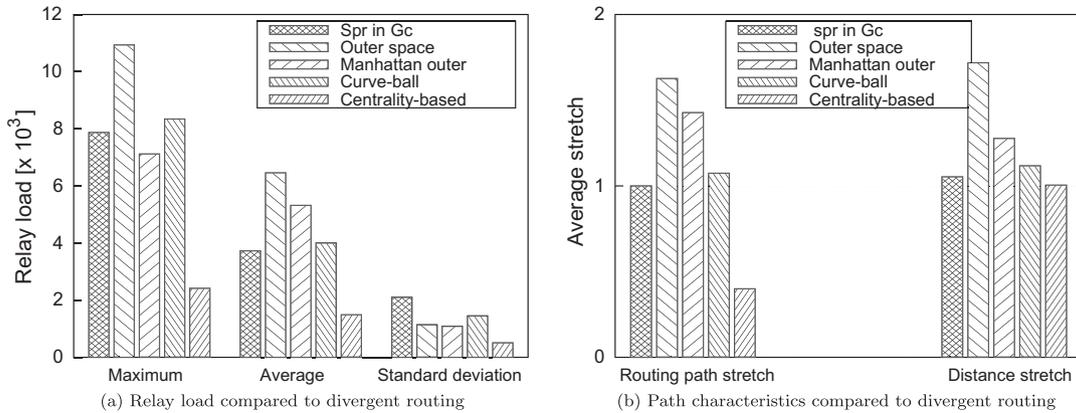


Fig. 5. Comparison of relay load and routing path characteristics of centrality power assignment and divergent routing.

deviation of relay load substantially without even increasing the average relay load. In most cases,  $\psi$  achieves up to 50% better load balancing than divergent routing schemes which is a useful practical result.

Fig. 5b shows comparison of average routing path stretch and distance stretch between the same set of schemes. Average routing path stretch can be defined as average ratio of hop-length of routing paths in any load balancing scheme to hop-length of the shortest routing path. Average distance stretch can be defined as average ratio of Euclidean length of a routing path (summation of length of all of its hops) to actual Euclidean distance between source and destination. This measures on an average how much routing paths of a scheme deviates from the straight line between the endpoints. All the divergent routing schemes increase path and distance stretch compared to SPR in Compow graph. Different from this,  $\psi$  actually reduces path stretch below one. This is because power control increases network connectivity which reduces number of hops required to be taken to reach the destination. Also,  $\psi$  reduces distance stretch when compared to Compow graph because increased connectivity makes end-to-end shortest paths deviate less from the straight line. It is generally believed that when divergent routing is used for load balancing, stretch factor and load balancing ratio are inversely proportional to each other [8]. As shown in [7], divergent routing strategy which has stretch factor of  $\sigma$  with compared to shortest path routing, can achieve load balancing ratio of  $O(\sqrt{n}/\sigma)$ . But when power control is used for load balancing, it is in fact possible to reduce stretch factor together with load balancing ratio, using shortest paths only.

As we saw that the centrality based power control yields improved load balancing with node-to-node traffic, we now discuss how the scheme performs with nodes-to-gateway traffic pattern. Fig. 6 shows the relay load of nodes when nodes send traffic to a central sink. When  $f = 1$ , all packets have to pass via the first tier nodes (neighbors of the sink), and hence their relay load is the maximum in the network. As the growth factor increases, nodes in the subsequent tiers are assigned high transmission power which allows them to jump over the first tier nodes. The relay load gets quickly balanced among nodes (lower standard deviation), and the overall behavior is similar to that in the case of node-to-node traffic.

4.2. Throughput capacity

Increasing power levels of nodes certainly results into better load balancing, but it also affects achievable spatial reuse and throughput. Increasing communication range and connectivity of nodes using power control results in longer links which causes interference in a larger area, reducing simultaneous transmissions.

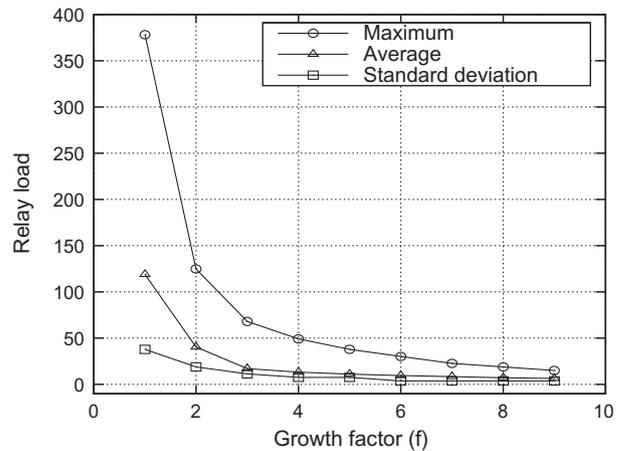


Fig. 6. Load balancing with  $\psi$  in uniform node-to-gateway traffic ( $n = 400$ ).

In this section, we show that centrality based power control also increases throughput capacity in random and clustered topologies under uniform node-to-node traffic. In case of uniform topologies such as a grid, centrality based power control shows a trade-off between load balancing and throughput capacity.

We use spatial reuse TDMA-based greedy link scheduler ([27,28]) for generating time slotted, conflict-free and feasible link transmission schedule. The end-to-end traffic demand between nodes is represented using a traffic demand matrix ( $T_R$ ). Once the shortest path routing is performed,  $T_R$  yields per-link transmission matrix ( $T_X$ ). We assume that there is a central controller entity which performs link scheduling. In the operation of greedy scheduler, first all links of  $T_X$  are sorted based on their interference score. Interference score of a link is the number of other links with whom the given link interferes and hence can not be scheduled simultaneously. Then scheduler chooses the first link in order to be scheduled in the current slot and tries to add more and more non-interfering links greedily until no more links can be added to the slot. The procedure repeats until all transmission requests of  $T_X$  are satisfied. [27] showed that such a scheduler has the time complexity of  $O(m \cdot n \cdot T)$ , where  $T = \sum_{i=0}^n \sum_{j=0}^n T_{X_{ij}}$ . If the total offered load  $G = \sum_{i=0}^n \sum_{j=0}^n T_{R_{ij}}$  and greedy scheduler requires  $S$  slots to schedule all the links, the network throughput is  $G/S$  traffic units per unit time. The TDMA scheme used here is centralized and we assume that there is tight synchronization between nodes, and resultant TDMA schedule is distributed to the nodes without any additional delay. Such an idealistic TDMA scheme is appropriate to study performance of centrality based power control in its most general form which is the central focus of the work.

We assume that simultaneous transmissions on two links  $uv$  and  $xy$  results into collision-free data reception at the receivers iff  $d_{ux}, d_{uy}, d_{vx}, d_{vy} > (\Delta \cdot d_{uv})$  and  $d_{ux}, d_{uy}, d_{vx}, d_{vy} > (\Delta \cdot d_{xy})$ , where  $d_{xy}$  is the distance between nodes  $x$  and  $y$  and interference ratio  $\Delta = 2$ . As an example, in coarse-grained TDMA, if every slot is 1 s and channel goodput ( $B$ ) is approximately 6 Mbps (as in 802.11b without RTS/CTS) in absence of interference, then resultant throughput with above mentioned TDMA scheduler is  $(G/S) * B$ .

It is important to note that heterogeneous power assignment using centrality leads to asymmetric links. The existence of asymmetric links can cause retransmissions at both MAC and transport layers and can result into duplicated data. For MAC layer, there are two ways of dealing with the problem. First way is to ignore the MAC layer acknowledgements and simply rely on transport layer to guarantee a reliable delivery. The presented scheme of centrality based power assignment can use this solution for implementation. Second, if there are MAC layer acknowledgements, we assume that the receiver of the link uses sufficiently high transmission power just to transmit an ACK to the transmitter. This allows the receiver to acknowledge the data reception even though its assigned power level as per the centrality strategy is lower. The resultant network graph after centrality based power assignment is directional due to non-uniform nature of power assignment. Even though routing is performed on asymmetric links, when these links are scheduled, we assume that receiver is using high transmit power to acknowledge the transmitter. This is justified further by the interference model presented above which also considers interference caused by the receiver's acknowledgement to other links and assumes that receiver is using transmitter's power level only for the purpose of acknowledgement. In the context of transport layer, when transmissions follow a single- (or few-) hop path from a node X to a node Y, but a multi-hop path from Y to X, resulting in delayed or lost acknowledgements and potentially multiple transmissions, hence duplications at the transport layer. This consideration is also important and we believe methods provided in [29–31] can be used to address it.

4.2.1. Uniform node-to-node traffic

First, we consider uniform node-to-node traffic pattern. Fig. 7 shows the aggregate network throughput for three different topologies. We choose a large number of distinct network flows ( $\approx \frac{n(n-1)}{2}$ ) where each flow carries amount of traffic which is uniform across all flows. It is important to note that we have examined the effects of different power level on the throughput under the same conditions of network load, but the results regarding their relative

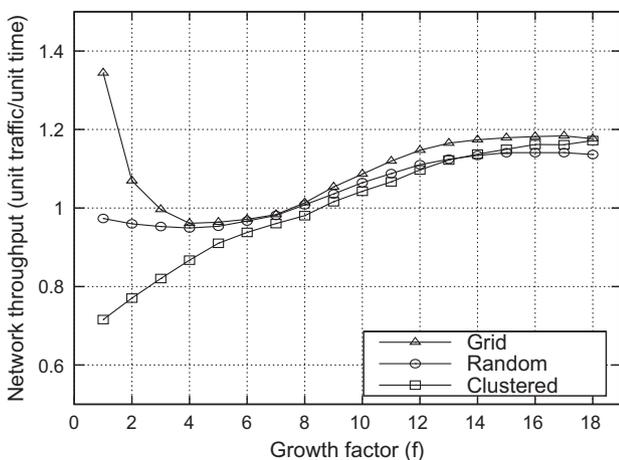


Fig. 7. Throughput capacity with centrality power control with node-to-node traffic ( $n = 400$ ).

performance are not dependent on any particular network load; thus results presented below hold for higher network loads as well – or any network load that is within the network capacity region. In case of grid, increasing the growth factor results into initial decrease of capacity followed by an increase. Even though load balancing improves with increase in growth factor, highest capacity is achieved at  $f = 1$  in case of grid topology. This is because highest spatial reuse is achieved at  $f = 1$  in case of grid and spatial reuse gradually decreases after that. On contrary, in case of random and clustered topologies, increasing growth factor also increases the throughput capacity along with better load balancing. As expected, random and clustered topologies achieve lower throughput compared to grid case. In random topologies, increasing power levels of certain nodes decreases the spatial reuse, but it also increases the connectivity among nodes. This results into many of the source nodes directly reaching the destination nodes and average routing path length decreases. The positive effect of reduced number of transmissions dominates the negative effect of reduced spatial reuse. This is especially a notable result because the reduction in relay load due to increased range more than offsets the reduction in spatial reuse due to increased interference, and hence overall throughput capacity improves.

In clustered topologies, inter-cluster links become traffic bottlenecks and they are required to be scheduled large number of times. This results into decreased throughput. Now, as the growth factor increases, more and more inter-cluster links are established which share the burden of previously bottleneck links and also improve the opportunity of spatial reuse. Due to better load balancing and improved spatial reuse, increasing growth factor in clustered topologies also increase the throughput. Since most of the real world topologies are either random or clustered, centrality based power control can yield good benefits of load balancing and capacity in them. Also, decision of choosing appropriate value of  $f$  depends on the network topology along with many other factors such as traffic pattern, traffic load, wireless radio characteristics such as maximum transmission power etc.

4.2.2. Uniform node-to-sink traffic

We now consider uniform node-to-sink traffic pattern. Fig. 8 shows the aggregate network throughput for three different topologies. In this traffic pattern, all nodes send data to the sink, and the amount of traffic in flows is uniformly distributed. As we can see that increase in the growth factor results into increased throughput in all three topologies.

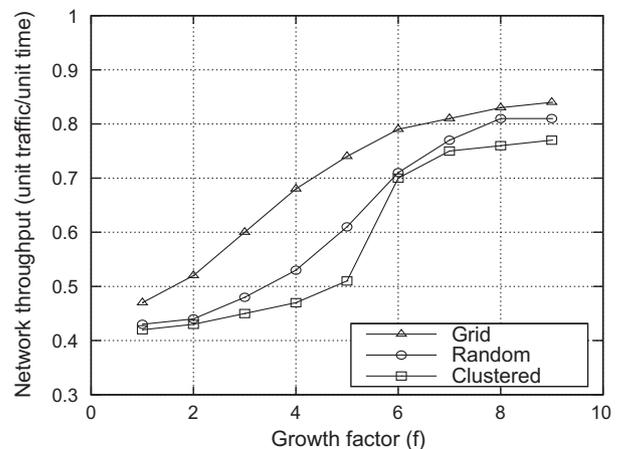


Fig. 8. Throughput capacity with centrality power control with node-to-gateway traffic ( $n = 400$ ).

As expected, the nodes around the sink have to perform the highest amount of relaying since all routing paths have to go through one of them. The links around the sink have to be scheduled a large number of times but they can not be scheduled in the same time slot since they mutually interfere. The resultant hot-spot reduces the network throughput due to disproportionate load balancing. When centrality based scheme is applied, nodes that were previously 2 hop or 3 hop neighbors of the sink (or even beyond that) can now directly reach the sink. This indeed decreases the spatial reuse due to creation and scheduling of long links, but the total number of required transmissions also decrease. Again, as it was in the case of node-to-node traffic, the decrease in required number of transmissions to be scheduled is faster than the decrease of spatial reuse which yields a net advantage in network throughput. The effect continues with increase of the growth factor until a point beyond which increasing the growth factor does not have any impact on network throughput.

A rather similar behavior is observed in the case of poisson-random topology. In the case of clustered topology, the increase of throughput is observed to be sharper in comparison with grid and random cases. This is attributed to the fact that for a given average inter-cluster distance, at certain value of  $f$ , the inter-cluster connectivity sharply increases which in turn also increases the spatial reuse (as it was in the case of node-to-node traffic) due to load sharing among inter-cluster links.

#### 4.3. Energy efficiency and network lifetime

In this section, we present the numerical results for energy efficiency of load balancing mechanisms and resultant network longevity. We model our sensor network where each sensor is a widely used MicaZ [32] node. Specifically, they utilize CC2420 [33] radio chip which offers as many as eight different transmission power levels. As described in [25], these Tx power levels have energy consumption in the form of  $P_T(d) = P_{T0} + P_A(d)$  where  $P_T(d)$  is the total power consumption to transmit at a distance  $d$  which contains a component  $P_{T0}$  independent of  $d$  and another component  $P_A(d)$  dependent on  $d$ . The transmission range independent component  $P_{T0}$  also has a significant impact on overall consumption because no matter at what power level a transmission occurs,  $P_{T0}$  is always accounted for all transmissions in the network. The actual power consumption (in mA) at eight discrete power levels are described in Table 1. The typical reception power consumption is 12 mA. It is obvious that the presented scheme has a crucial dependence on actual signal propagation model and resultant communication range. As mentioned before, we do not assume a specific signal propagation model. Instead, for simulations, we rely on real-world measurements of MicaZ listed in [34] for the relationship between transmission power and corresponding communication range (listed in Table 1) in standard outdoor environment. The maximum transmission range ( $d$ ) achievable at various Tx power levels is determined in such a way that reception at distance

$d$  has above 95% packet reception rate [34]. It is assumed that power consumed in transmission and reception functions dominates the consumption of all other tasks and ideal MAC protocol is employed for low-level implementation. For simulations, continuous power levels resulting from centrality based power assignment are mapped to nearest higher discrete power level. Each node is powered by two AA batteries (2000 mAh, 3 V) and transmission of packets are assumed to be 500 ms long.

Fig. 9a shows global energy expenditure with various load balancing schemes when every node in the network sends one packet to every other node in the network. As explained before, any divergent routing scheme consumes more energy than SPR in  $G_C$  because of increased path lengths. On the other hand,  $\psi$  reduces the total number of transmissions and receptions (path length) which results into reduced energy consumption. This is in line with results presented in [25] which shows that routing on shortest paths in a more connected network (lesser number of hops) is more energy efficient.

##### 4.3.1. Uniform node-to-node traffic

We consider two measures of network lifetime in the case of uniform node-to-node traffic pattern: time to death of the first node and loss of coverage. These measures are discussed in [35] and are widely used in sensor network research. In both the measures, after certain number of messages are transferred between nodes, a node in the network ends up depleting its battery which is marked as the death of the first node. This first node is highly likely to be located near the center in the case of SPR in  $G_C$  and often has to perform maximum amount of relaying. Fig. 9b shows comparison of death of first node based network lifetime in SPR in  $G_C$ , curve-ball routing and centrality based load balancing. Divergent routing schemes like curve-ball improves on time to the first death by better distributing the relay load in the network. The  $\psi$  improves the lifetime significantly because it achieves better load balancing than other mechanisms.

In the second measure of lifetime, it is assumed that every sensor has a role of sensing certain number of *event points*. Every point is assumed to be covered by approximately 15 sensors. This way, when co-located 15 sensors die, a particular event point becomes uncovered, resulting into dysfunctional state of the network. This is different from the first measure because network can still continue performing its work even after the death of first node. As can be observed in Fig. 9b that  $\psi$  achieves significantly longer lifetime because of better load balancing and reduced path lengths. As before loss of coverage occurs near the center for SPR in  $G_C$  and near the area between periphery and center in curve-ball routing. It was observed that in the case of  $\psi$ , loss of coverage happens almost uniformly randomly in the network, which demonstrates improved load balancing.

##### 4.3.2. Uniform node-to-sink traffic

Now we consider the uniform node-to-sink traffic pattern which is a more practical case for real world sensor deployments. Since all nodes are sensing and transferring their packets to the sink, disconnection of the sink from the nodes is a useful and accurate network lifetime measure [1]. Specifically, it was shown that when all the nodes providing connectivity to the sink (first tier nodes) exhaust their battery levels, sink can no longer be reached and the network becomes dysfunctional. Here it is also assumed that sink itself is not power constrained. When the sink is placed at the center of the network, there is no significant benefit of divergent routing because all the packets have to eventually traverse through the first tier nodes only. Hence, only SPR is considered here for comparison. In a typical lifetime of such a network, all nodes start by sending packets to the sink and nodes in first few tiers start depleting their batteries very quickly. At a certain point,

**Table 1**  
MicaZ specifications [33,34]: transmission power levels, their power consumption and communication range.

Transmission power level	Power consumption (mA)	Communication range (meters)
1	8.5	10
2	9.9	15
3	11.2	25
4	12.5	35
5	13.9	45
6	15.2	50
7	16.5	55
8	17.4	65

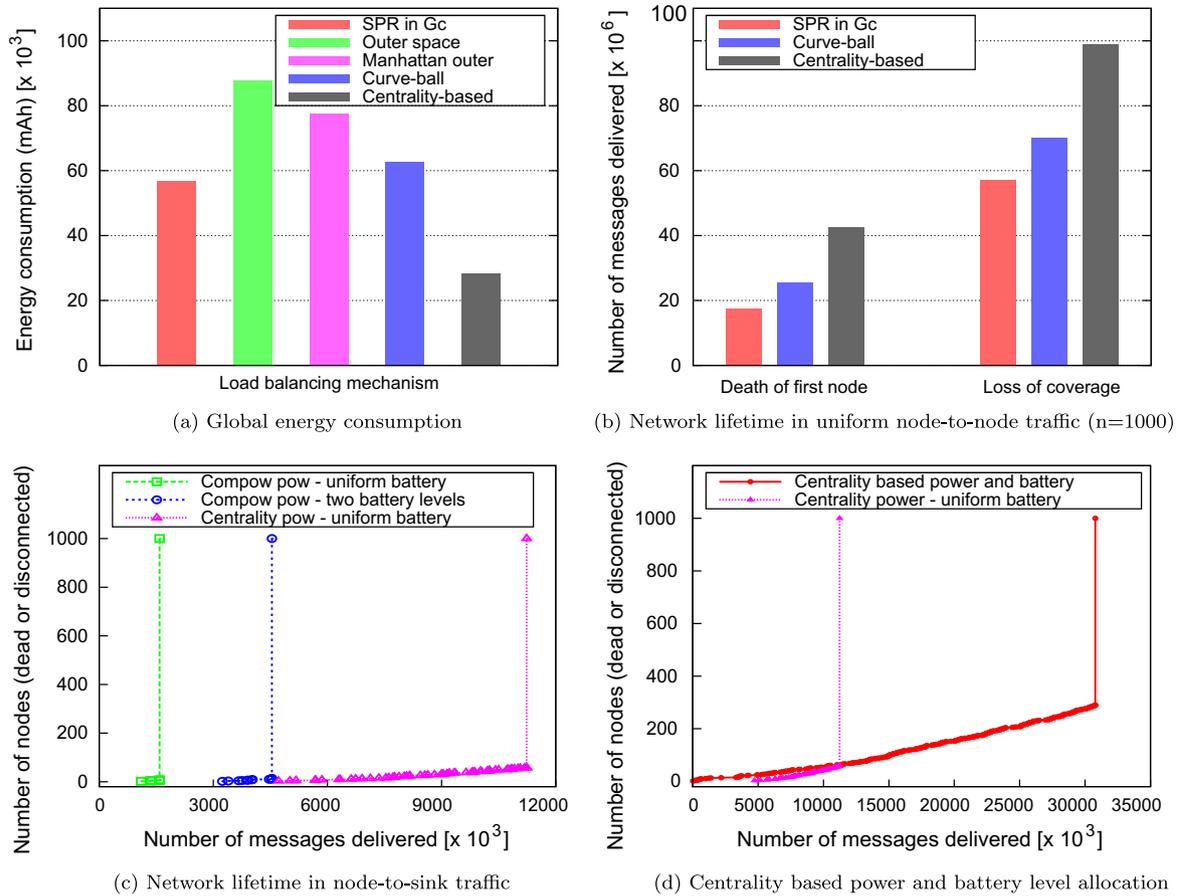


Fig. 9. Effects of centrality based power control on load balancing, energy efficiency and network lifetime.

a node, most probably in the first tier, dies but other nodes of the first tier still provides sink connectivity. As an when more and more first tier nodes die, it increases the relay load of remaining first tier nodes since all shortest paths to sink now pass through them. Eventually, all nodes in the first tier deplete their batteries and suddenly entire network becomes disconnected. Note that we restrict our attention to single sink case here. In the case where there exists multiple sinks, and nodes can switch to a different sink depending on the connectivity is beyond the scope of this work. This is because such a switching changes the traffic pattern which in turn requires recalculation of centrality values and re-assignment of transmission power. Such a dynamic strategy and resultant protocol is part of our future work (Section 5).

The case of single sink is displayed in Fig. 9c where all nodes ( $n = 1000$ ) are assumed to be operating at Compow power level and have uniform battery levels too. Fig. 9c depicts number of nodes that can not reach the gateway because either they have depleted their batteries or they are disconnected from the sink due to deaths of other nodes. Initially, most of such depicted nodes are the ones who have depleted their batteries and are mostly in first tier. Once all the nodes of first tier die, all remaining nodes suddenly become disconnected (vertical straight transition of the curve) and network lifetime has ended. It is worth noting that at this point nodes who are disconnected but yet not dead, have not even depleted half of their batteries in most cases.

To mitigate the problem, [1] presented an approach where nodes near the sink (first few tiers) are assigned higher battery levels than other in such a way that global battery budget remains unchanged. In the simplest case of their solution, there are two battery levels in the network where 13% of total nodes (near the sink) have 5718 mAh batteries while rest of 87% have batteries

with capacity of 1442 mAh. Now, nodes who are responsible for relaying more packets are assigned higher battery levels to live longer and increase network lifetime. We simulate this case and results are presented in Fig. 9c which shows that even with just two battery levels, overall network lifetime can be about three times longer than the base case. The third case presented in the Fig. 9c is the case of load balancing with centrality based power control. Here, the battery levels of the nodes remain uniform but as before, nodes which are likely to relay more packets for others are assigned higher power levels. This way, as we move from the periphery towards the sink in the center, power levels of the nodes increase in every tier continuously. This allows the nodes away from the sink to jump over the first few tiers' nodes and directly reach the sink which reduces their relaying burden. Hence, relay load of nodes are better distributed among the nodes even in case of node-to-sink traffic pattern (not possible with divergent routing schemes). This results into deaths of nodes which are relatively more consistent over time (Fig. 9c) and significant increase of network lifetime is observed.

Betweeness centrality can also be used to assign battery levels to the nodes since the nodes which are likely to perform more relaying are actually the nodes which deplete their batteries before others. Fig. 9d shows the lifetime behavior of a network where nodes which are likely to relay more packets (in first few tiers) for others are assigned higher power levels as well as higher battery levels. The same algorithm presented in Section 3.2 is also used to perform battery assignment in a way that overall battery budget remains unchanged. Different from discrete multiple battery level case, here the battery levels of the nodes change continuously. As can be observed in Fig. 9d, such a power and battery assignment results into substantially longer network lifetime.

The lifetime is even better than the case where only power levels are assigned using centrality but the battery levels are uniform. Though the death of the first node occurs earlier than other cases, an interesting observation is that node deaths and disconnections are more consistent over the time and uniform across the network which shows improved balance between relay load distribution and battery resource allocation.

## 5. Conclusions and future work

In this paper, we proposed centrality based selective power control scheme which achieves better relay load balancing compared to other divergent routing schemes. We showed that centrality based power control significantly increases the network lifetime of a sensor network. Since the current scheme is centralized, we are devising a distributed protocol based on CSMA-CA as part of our ongoing work. In the distributed protocol, nodes periodically share their centrality values based on queue lengths of their backlogged queues (current relay load) with their neighbors. Much as OSPF protocol, this information can be further distributed and every node can receive or calculate the global relay load information. Even though this dissemination might work slowly, once this information is received it can be gainfully utilized for a longer period at least in case of static networks. The other possibility that we are currently exploring is to yield faster but possibly less accurate estimation of relay load. In this strategy, every node disseminates its relay load information only in its collision neighborhood. In both cases, the relay load information (global or local) is utilized to calculate the transmission power of nodes such that nodes with higher relay load is assigned proportionally high power level. This will allow a complete design and analysis of a distributed and dynamic power control protocol based on betweenness centrality. Note that there are many challenges associated with this such as addressing issues like generation of asymmetric links due to non-uniform power assignment of centrality mechanism. We plan to accommodate such issues in our ongoing work.

### Appendix A. The maximum edge weight of any Euclidean minimum spanning tree of a set of nodes is the Compow weight of the set of nodes

**Proof.** Let  $V$  be a set of nodes placed on a plane using an arbitrary distribution. Let  $G_U = (V, E, r)$  denote a unit disk graph on  $V$ . There exists an edge from node  $u$  to node  $v$  if their Euclidean distance  $d_{uv} \leq r$ . The weight of an edge in  $E$  is the Euclidean distance between the endpoints of the edge. Now, let  $G_c = (V, E, r_{min})$  denote the Compow graph of  $V$  where  $r_{min}$  is the minimum Euclidean distance such that  $G_c$  has only one connected component (all nodes in  $V$  are connected). We are interested in proving that  $r_{min}$  is the maximum edge weight of any Euclidean minimum spanning tree on  $V$ .

The proof proceeds in two steps. In the first step, it is shown there exists a minimum weight spanning tree in the Compow graph  $G_c$  which is also a minimum weight spanning tree on  $V$ . In the second step, it is shown that any minimum spanning tree on  $V$  also minimizes the maximum edge weight across all minimum spanning trees.

*Part 1:* By definition,  $G_c$  contains a minimal set of edges ( $E_c$ ) such that weights of all edges are no more than minimum required to remain connected. This leads to two characteristics of  $G_c$ . First, all edges of maximum weight in  $G_c$  form a *disconnecting set* of edges whose removal will cause disconnection of  $G_c$ . Second, all minimum spanning trees in  $G_c$  have at least one of the maximum

weight edges. We use these two properties to prove that there exists a minimum weight spanning tree in the Compow graph  $G_c$  which is also a minimum weight spanning tree on  $V$ .

Let  $T_c$  be a minimum weight spanning tree in  $G_c$  and  $T$  be a minimum weight spanning tree on  $V$ . We show first that there exist  $T$  and  $T_c$  such that maximum edge weight  $\max(T) = \max(T_c)$ . Then it is shown that  $T_c$  is also a minimum weight spanning tree on  $V$ . (Proof by contradiction) If  $\max(T) > \max(T_c)$ , there exists an edge  $e_c \in G_c$  which can connect the end points of edge in  $T$  with weight  $\max(T)$ , yielding new minimum weight spanning tree (say  $T'$ ) on  $V$ . The overall weight of  $T'$  is lesser than  $T$ , which contradicts with the fact that  $T$  is a minimum weight spanning tree. So,  $\max(T) \not> \max(T_c)$ . Now, if  $\max(T) < \max(T_c)$ , there exists a lower weight edge which can connect the endpoints of edge with weight  $\max(T_c)$  in  $G_c$ . If so,  $T_c$  can not be minimum weight spanning tree in  $G_c$  and set of maximum weight edges also do not form a disconnecting set in  $G_c$ . Based on this contradiction,  $\max(T) \neq \max(T_c)$ . Hence,  $\max(T) = \max(T_c)$ . Since all edges in  $G_c$  are of lesser or equal weight than  $\max(T_c)$ , there exists a  $T$  on  $V$  such that  $T = T_c$ . This way, there exists a minimum weight spanning tree in the Compow graph  $G_c$  which is also a minimum weight spanning tree on  $V$ .

*Part 2:* Any minimum spanning tree on  $V$  also minimizes the maximum edge weight across all minimum spanning trees.

Proof (by contradiction) – Suppose  $T_1$  is a minimum spanning tree on  $V$ . Also suppose that there exist another minimum spanning tree  $T_2$  which minimizes maximum edge weight. Since  $T_1$  does not minimize the maximum edge weight, there must be an edge (say  $e_1$ ) in  $T_1$  which is of higher weight than all edges in  $T_2$ . Removing  $e_1$  from  $T_1$  will create a forest with exactly 2 trees. Now we must find an edge (say  $e_2$ ) in  $T_2$  which can connect two trees of  $T_1$  forest. Since  $e_1$  was of higher weight than all edges of  $T_2$ , new connected tree  $T_1$  which contains  $e_2$  has total lower overall weight. This contradicts with the fact that  $T_1$  was a minimum spanning tree. Hence, proved that  $T_1$  also minimizes maximum edge weight in all minimum spanning tree. (or put another way, maximum weight edge in any minimum spanning tree is also maximum across all minimum spanning trees). This can also be stated as, let  $\max(MST)$  denote the maximum edge weight in a minimum spanning tree, there will be at least one edge in all minimum spanning trees which has weight  $\max(MST)$ .

Since every minimum spanning tree on  $V$  has an edge with weight  $\max(MST)$ , and  $G_c$  contains a minimum spanning tree of  $V$ , it is proven that the maximum edge weight of any minimum spanning tree of a set of nodes is the Compow weight of the set of nodes.  $\square$

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