Impact of Power Control on Capacity of Large Scale Wireless Mesh Networks *

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***Abstract*—Wireless Mesh Networks (WMNs) have emerged as low-cost alternative for broadband access networks in metropolitan area. In this paper, we revisit power control as a network layer problem in design of WMNs. Specifically, we show that increasing power levels of nodes in WMNs actually results into throughput improvements in node-to-gateway traffic pattern. It is proven that WMNs can achieve per node throughput of $O(1/\delta n)$ where δ is factor dependent on hop-radius of the network, which in turn is reliant on power control. Also, it is shown that in case of multiple gateways, increasing power levels of nodes is more cost-effective because it results into increased throughput with fewer number of gateways.**

I. Introduction and Problem Description

WMNs are being deployed as *last few* miles wireless access networks for ubiquitous network connectivity. Transmission power control has been well-investigated for multi-hop ad-hoc networks but its advantages are still not completely explored in WMNs where majority of the traffic flows between the nodes and the gateways which provide Internet connectivity. It is generally acknowledged that decreasing transmit power level of nodes to the minimum required to retain network connectivity achieves optimal network capacity in the case where traffic flows between any random pair of nodes. Two contradicting power control strategies are proposed by *short-range-multi-hop* Compow [1] and *long-range-single-hop* DirectTrans [2]. In Compow, all nodes use a uniform constant power level which is minimum required to maintain network connectivity (sparse topology). Compow achieves better concurrency in link scheduling but requires more relaying at nodes with longer routing paths. In sharp contrast to this, in DirectTrans, nodes willing to transmit increase their power level until receiver can be reached in single hop (fully connected topology). DirectTrans benefits from fewer transmissions and no relaying but suffers with lower spatial reuse due to long high interference links. In this paper, we investigate the effects of power control in WMNs with node-to-gateway traffic pattern. Specifically, we address the question - what is the impact on throughput when transmission power of nodes are increased from Compow to DirectTrans with node-to-gateway traffic?

In contiguous work (currently in submission), we have shown that power control has a beneficial effect on the balancing of relay load in multi-hop networks. However, it might appear that such gain comes at the cost of reduced throughput, because of increased interference; in this paper, we focus on that issue and show that the effect on throughput is also beneficial, though to a lesser extent in some cases. The contributions of the paper are as follows - First, using analysis and simulation results we show that increasing power level of nodes to maximum possible always results into better throughput in case of node-to-gateway traffic pattern. It is proven that per-node throughput of $O(1/\delta n)$ can be achieved in WMNs with n nodes where δ is a factor dependent on worst case hop distance from any node to the gateway. Increasing power level of nodes reduces the value of δ which results into throughput improvements. Next, we show that when nodes increase their power levels, required number of gateways can be reduced by a certain percentage without sacrificing the throughput. That is, for any given number of gateways better throughput can be achieved with increase of power levels.

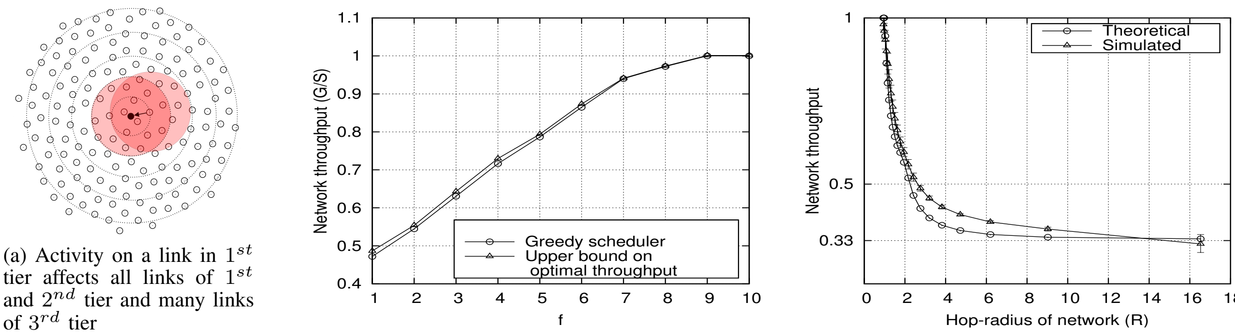
II. Network Model and Preliminaries

We model the network graph using unit disk graph $G_U = (V, E, r)$ where V is the set of nodes and for any two nodes u and v, there exists an edge $uv \in E$ if their Euclidean distance $d_{uv} \leq r$. Compow range (r_{min}) is defined as minimum value of r such that G_U is connected. Similarly, DirectTrans range (r_{max}) is minimum value of r such that G_U is fully connected. To understand the impact of power control, we uniformly increase power level of all nodes step-by-step such that at any step, all nodes operate at the same power level and hence same transmission range. Increase of power levels can actually be interpreted as bounding the maximum power level of nodes. That is, if a node increases its power level, this does not mean that it will always transmit at new increased power level. If a neighbor is reachable at a lower power level, it will utilize that to communicate with it. We do not assume any specific signal propagation model because power level of node are presented in terms of their communication range. As an example, in Compow, all nodes operate at power level $P(r_{min})$ which is necessary and sufficient to achieve communication range of r_{min} at all nodes. Now, if a node wants to increase its communication range by a factor of f, it tunes its power level to $P(f \cdot r_{min})$. This way, increase of power levels are normalized to the Compow range r_{min}, not to Compow power level $P(r_{min})$ because $P(f \cdot r_{min}) \neq f \cdot P(r_{min})$ necessarily.

We use uniform node-to-gateway traffic pattern where all nodes send uniform amount of traffic to the gateway (consideration of uniform gateway-to-node traffic does not change implied results). All nodes operate on same channel and shortest paths are used to route the data. TDMA-based greedy scheduler is used to generate time slotted and conflict-free link transmission schedule. Once routing is performed, end-to-end traffic demand matrix (T_R) yields per-link transmission matrix (T_X). We assume that a central controller performs greedy scheduling in which all links of T_X are first sorted based on their interference scores (number of other links in T_X it interferes with). Then scheduler chooses the first link in order to be scheduled in the current slot and tries to add more and more non-interfering links greedily until no more links can be added to the slot. The procedure repeats until all transmission requests of T_X are satisfied. Such a scheduler was presented for physical interference model in [3] and was shown to have time complexity of $O(m \cdot n \cdot T)$, where m is the number of edges and $T = \Sigma_{i=0}^{n} \Sigma_{j=0}^{n} T_{X_{ij}}$. If G is the total traffic demand and greedy scheduler requires S slots to schedule it, the resultant network throughput is G/S.

Any communication on link uv causes interference in area of two partially overlapping disks centered at the endpoints.

*This work is supported by the U.S. Army Research Office (ARO) under grant W911NF-08-1-0105 managed by NCSU Secure Open Systems Initiative (SOSI). The contents of this paper do not necessarily reflect the position or the policies of the U.S. Government.



(a) Activity on a link in 1^{st} tier affects all links of 1^{st} and 2^{nd} tier and many links of 3^{rd} tier

(b) Greedy scheduler performs near-optimally in WMNs

(c) Throughput and hop-radius in WMNs

Fig. 1: Effect of power control on link scheduling in wireless mesh networks

Radius of such a *double disk* is $(\Delta \cdot d_{uv})$ where $\Delta = 2$ is the interference ratio. Simultaneous transmissions on two links uv and xy results into collision-free data reception at the receivers *iff* $d_{ux}, d_{uy}, d_{vx}, d_{vy} > (\Delta \cdot d_{uv})$ and $d_{ux}, d_{uy}, d_{vx}, d_{vy} > (\Delta \cdot d_{xy})$. Such a consideration of interference in greedy scheduler allows us to focus on network connectivity without delving into physical layer details.

III. Power Control

It was proven in [4] that per node throughput in WMNs can not be more than $O(1/n)$ which is significantly worse than seminal result of $O(1/\sqrt{n})$ [5]. This is because traffic of every node, no matter how many hops away it is from the gateway, has to ultimately pass through bottleneck links connecting to the gateway. Per node throughput of $O(1/n)$ is also achievable in WLANs but it has been empirically observed that WMNs often achieve even worse throughput than WLANs (often by a factor of 2 or 3). We show next that WMNs in fact achieve per node throughput of $O(1/\delta n)$ where $\delta \geq 1$ is a factor dependent on the hop-radius of network graph. Hop-radius is defined as length of the longest path from a node to the gateway assuming that the gateway has the least eccentricity among all nodes.

A. Single Gateway

1) Analysis: We assume that the nodes are distributed with uniform density on circular network area with radius R_N (Fig. 1a). Let λ denote the Compow distance and power level of all nodes are incremented using factor f which results into communication range of $f\lambda$ at every node. Let tier t be a set of nodes that are t hops away from the gateway and R be the number of tiers. With uniform density, $R \approx \frac{R_N}{f\lambda}$ and number of nodes in tier i, $n_i = \frac{(2i-1)n}{R^2}$. If every node sends p packets to the gateway, links of any tier i have to forward traffic of all nodes at tier $i...R$. This way, load of all links in tier i, $L_i = p \cdot (n - \sum_{j=1}^{i-1} n_j) = p \cdot \frac{n(R^2-(i-1)^2)}{R^2}$.

Now, *Bottleneck Collision Domain* (BCD) is defined as set of mutually interfering links in which no more than one link can be scheduled in same slot and cumulatively they have to transmit maximum total traffic. It is easy to see that in WMNs, such a BCD is created near the gateway. When a link in first tier is scheduled, almost all links in second and third tiers suffer from interference and can not be scheduled concurrently. For pessimistic worst case analysis, we consider that all links of first three tiers are part of a BCD. Now, the size of BCD ($\mid BCD \mid$) is defined as total amount of traffic its links have to transmit which can be calculated as:

$$\mid BCD \mid = L_1 + L_2 + L_3 = p \cdot n \left(\frac{3R^2 - 5}{R^2} \right) \quad (1)$$

By definition of BCD, total traffic $G = np$ requires at least $\mid BCD \mid$ number of slots. Hence, network throughput can not be more than $\frac{np}{|BCD|}$ and with assumption of absolute fairness, per node throughput can not be more than $\frac{p}{|BCD|}$. Using (1), per node throughput can not be more than $O(1/\delta n)$ where $\delta = \frac{3R^2-5}{R^2}$, a factor dependent on R.

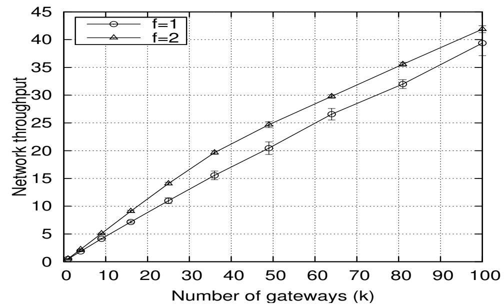
When $R = 2$, every node can achieve throughput of $\frac{p}{|BCD|}$ where $\mid BCD \mid = L_1 + L_2 = p \cdot n \left(\frac{2R^2-1}{R^2} \right)$ and hence $\delta = 1.75$. Similarly, it can be shown that when $R = 1$, $\mid BCD \mid = L_1 = p \cdot n$ and fraction $\delta = 1$. This results into $O(1/n)$ where every node can directly reach gateway similar to the WLAN case. Surprisingly, factor δ converges to 3 as $R \rightarrow \infty$ in (1). This suggests that no matter how large the hop-radius of the network becomes, WMNs can still achieve one third of the capacity of WLANs even in the worst case. This is due to the fact that at least one link in first three tiers can always be scheduled in almost all slots.

2) Simulation Results: In node-to-gateway uniform traffic pattern, greedy scheduler performs near-optimally because almost all links away from the gateway (beyond first three tiers) can be almost always be scheduled in parallel with links of first three tiers (BCD). Hence, number of slot required $S \approx \mid BCD \mid$. This is verified by comparing the performance of greedy scheduler and upper bound of optimal throughput (Fig. 1b). The upper bound on optimal throughput is derived using conflict graph. Conflict graph is created by placing a vertex for every link of communication graph and an edge between two vertices if corresponding links in communication graph interfere with each other. Traffic demand of every link in communication graph is used as weight of the corresponding vertex in conflict graph. Now, maximum weight *clique* in conflict graph is a set of links in communication graph which has maximum total traffic but only one of which can be scheduled in any slot even with optimal scheduler. Fig. 1b shows that increasing f increases the network throughput consistently. Similarly, Fig. 1c verifies simulated throughput with analytical result of $O(1/\delta n)$. For simulations, 1000 nodes are randomly placed on a unit-disk with the gateway placed at the origin. For achieving continuous values of throughput points, we assume hop-radius $R = \frac{R_N}{f\lambda}$ to be continuous also. As the hop-radius of the network increases network throughput drops from 1 and converges to 0.33 for large values of radius. It is noticeable that even when radius increases from 1 to 2, network throughput drops significantly (by a factor $\delta = 1.75$). The minor difference between the analytical and simulated curve can be attributed to the fact that average spatial reuse in first three tiers is often slightly larger than 1. This is in line with our rather pessimistic analysis which assumed that only one link can be scheduled in first three tiers. If it would have been assumed that only one link can be scheduled in first *two* tiers, analytical value of throughput would have converged to 0.5 (Fig. 1c) which would have been clearly too optimistic when compared to actual simulated results. The analysis shows clear dependence of δ on R which in turn is dependent on f. Dependence on f shows that increase of f results in smaller network hop-radius R which reduces factor δ, resulting into increase of throughput. This suggests that it is always better to increase power level of nodes which decreases the network hop-radius and increases the throughput. This is a useful capacity result for WMNs since it proves that reducing worst case hop distance to gateway always performs better in terms of throughput.

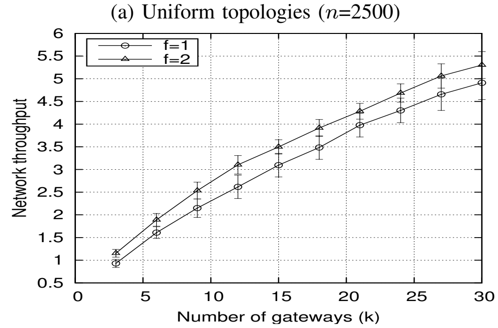
B. Multiple Gateways

In large metro-scale WMNs, having one gateway is not scalable and sufficient to meet the demand. When more and more nodes are assigned the role of gateway, network throughput increases but associated cost also increases since such gateway nodes typically have a high-speed fiber link or a satellite connection. In such case, objective of network designer is to achieve as much throughput as possible with fewer number of gateways. We saw that increasing power level of nodes results into better throughput in one gateway case. In this section, we show that this is also true for multiple gateways and in fact power control can be used to achieve better throughput with lesser number of gateways.

Problem of placing k gateways in the network can be formulated as facility location problem or problem of finding $k-$medians of network graph. Since these problems are known to be NP-hard, we use simple yet efficient heuristics to place the gateways. In uniform random topologies, square network area is divided into a grid and the nodes nearest to the grid intersection points are assigned the role of gateways. Such a placement allows each node to equally share the gateway capacity when every node forwards its data to the nearest gateway using shortest paths. Also, number of nodes which send data to each gateway is almost equal, resulting into fair load balancing among gateways. We simulate the scenario using 2500 randomly distributed nodes with number of gateways increasing from 1 to 100. As before, power control is used to increase the transmission range of nodes by factor of f and greedy scheduler is used to schedule the link transmissions. Fig. 2a shows impact on throughput with different values of f when number of gateways in the network are increased. As can be seen, when nodes double their transmission range, on an average 20% more throughput can be achieved for any given number of gateways. This is a cost-effective solution since better throughput can be achieved with lesser number of gateways when nodes increase their power levels. After a certain value of f ($f \geq 3$ in our scenario), power control advantages become insignificant since at higher power levels, BCD of different gateways start overlapping with each other. Such value of f is still very high when Compow range is relatively smaller compared to network span and there is enough opportunity to increase power levels in such cases.



(a) Uniform topologies (n=2500)



(b) Clustered topologies (n=300)

Fig. 2: Power control in WMNs with multiple gateways

Grid-based gateway placement can not be used in non-uniform clustered topologies because only few of the grid intersection points may actually lie within the clusters. For different topologies, same grid placement might result into different number of gateways. Instead we use a simple heuristic where k gateways are equally divided into clusters which are then randomly chosen in each cluster. We simulate the scenario using 300 nodes where number of gateways are increased from 1 to 30. Fig. 2b shows that even in case of non-uniform topologies, increasing power level of nodes results into upto 15% throughput improvement. Since optimal gateway placement would result into better load balancing among gateways than random placement, power control advantages are likely to be more because better spatial reuse can be achieved among the bottleneck links near the gateways.

IV. Conclusions and Future Work

In this paper, we showed that increasing power levels of nodes results into throughput improvements in WMNs. In case of single gateway, WMNs can achieve per node throughput of $O(1/\delta n)$ where δ is a factor dependent on hop-radius of network. Such power control is also cost-effective in case of multiple gateways since it is possible to achieve better throughput with fewer number of gateways in WMNs. It is our ongoing research to investigate effects of power control with non-uniform traffic (also intra-mesh traffic) and its evaluation using implementation on our 802.11 wireless testbed.

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